Corruption in Multidimensional Procurement Auctions Under Asymmetry

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August 8, 2019

Abstract

We examine corruption in first- and second-score procurement auctions in an asymmetric bidder setting. We assume that the auction is delegated to an agent who possesses more information about quality than the procurer and is known to be corrupt with some probability. Using this information asymmetry, the corrupt agent asks for a bribe from one of two bidders and promises to manipulate bids in return. We show that the agent approaches the weaker firm for higher levels of bidder asymmetry in both the auction formats. Using a symmetric quasi-linear scoring rule we show that neither the first- nor the second-score auction implements the optimal mechanism, with or without corruption. Our numerical simulations suggest that the buyer prefers the first-score auction when the stronger firm is approached by the agent in the second-score auction. If the weaker firm is favored on the other hand, the buyer switches to the second-score auction when the probability of corruption is high. Finally, our paper highlights the limited manipulation power of the agent in the second-score auction.

Keywords: scoring auctions, asymmetric bidders, corruption, public procurement.
JEL classification: C72; D73; H57; K42.

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1 Introduction

The purpose of this paper is to examine corruption in a multidimensional procurement auction with asymmetric bidders. We study procurement auctions in which the bidders endowed with unidimensional type spaces submit multidimensional bids in the form of price-quality tuples. These tuples are then converted into a unidimensional score using a quasi-linear scoring rule. This procedure, which is used to buy differentiated products, is called a scoring auction. Procurement agencies which use such scoring auctions include the Department of Defense (DoD), State Departments of Transportation (DOTs) in the US and the Ministry of Land, Infrastructure and Transportation in Japan.

While auctions are generally considered to be an efficient mechanism for awarding procurement contracts, a separation between the auctioneer (a public official or agent) and the buyer (government agency or principal) gives rise to the possibility of corruption. Such a separation occurs whenever the buyer delegates the auction to an agent who possesses superior information about quality of the product. The agent in charge of verification of the quality of the good can, in lieu of a bribe, favor one of the firms and allow it to supply a lower quality than promised.

We study corruption in 'first-score' and 'second-score' auctions with two asymmetric bidders using a model similar to that of Celentani and Ganuza[1]. We assume that the efficiency types of both firms are drawn from a uniform distribution and that the strong seller's type distribution is a "stretched" version of that of the weak seller. While the procurer is aware that the agent is corrupt with some (exogenously given) probability, the corruption arrangement is endogenous, as the dishonest agent approaches one of the two firms and makes a take-it-or-leave-it bribe offer. In exchange, the agent promises to manipulate bids in favor of the (potential) accessory and to allow that firm to produce a lower quality, q_C. As the agent manipulates bids, he portrays the favored bidder to be more efficient than that of his rival, and in order to avoid detection, ensures that the total expected payment made by him is equal to the one made by his honest counterpart. The auction proceeds as in the honest setup if the offer is rejected.

In this setting, we attempt to answer the following questions:

1. Given that the bidders are asymmetric, which firm does a corrupt agent approach for a bribe? The answer to this question *could* determine the winner of the procurement auction.

2. What is the optimal mechanism for such multidimensional procurement auctions with asymmetric bidders?

3. Is it possible to implement the optimal mechanism using either a first-score or second-score auction? If the answer is in the negative, what is the preferred scoring rule for the buyer for a given probability of corruption and level of bidder asymmetry?

4. Are there conditions under which such an auction (with corruption) is efficient?

Celentani and Ganuza[1] study the impact of competition on corruption in a multidimensional procurement auction identical to ours and show that the first-score auction implements the optimal auction in a symmetric bidder setting. However, the symmetry assumption is violated in many real-life environments as firms are observed to be asymmetric due to differences in size, location, processing capacity, and/or technology, in addition to their position in the market (incumbent or entrant). Since a general form solution is not possible in the presence of asymmetry, we, therefore use specific functional forms in order to answer our research questions. We assume that due to legal constraints, the buyer uses a symmetric scoring rule while evaluating the submitted bids. Our assumption is supported by legal precedents which prohibit favoritism in government procurement processes on the basis of race, sex or ethnicity[1].

The preferred scoring rule of the buyer is related to the bidder who is approached for a bribe by the agent and the corresponding manipulation powers of the agent. We show that while the agent is able to manipulate bids freely in the first-score auction (and in both the formats in the symmetric bidder setting), he is unable to do so in the second-score auction. In particular, the agent is unable to rig bids in favor of the weaker bidder whenever the efficiency parameter of his opponent is higher than the maximum possible efficiency of the preferred bidder. The second-score auction, therefore, is more effective in countering bid manipulation in the asymmetric bidder setting.

We solve for the optimal auction, without and with corruption, and show that both require the buyer to penalize the stronger bidder (Propositions 3 and

[1]In the United States, for example, the California Civil Rights Initiative (Proposition 209, 1996) ended race- and gender-based programs which were meant to promote participation in public contracting. A similar initiative was passed in Washington in 1998 (Initiative 200). In addition, auction designs that explicitly favored some groups, have been successfully challenged in the US Supreme Court (eg. Adarand Constructors, Inc. vs Peña, 1995).

7). We also show that the limited manipulation power of the agent in the asymmetric setting has important implications for the optimal auction with corruption. Notably, it requires the buyer to deduce the bidder who will be approached for a bribe and to appropriately alter the weight that is assigned to the quality component of the bid based on the identity of the winner. As in unidimensional asymmetric auctions, we find that neither the first- nor the second-score auction implements the optimal mechanism.

We find that the agent approaches the weaker firm for a bribe for higher levels of bidder asymmetry in both the first-score and second-score auction and that the corresponding incidence falls with the minimum level of efficiency (Results 1 and 2). This is due to the expected profit from the honest mechanism (outside option), which rises faster for the stronger bidder as the level of bidder asymmetry increases and subsequently reduces the bribe that can be extracted from him by the agent. The minimum level of efficiency on the other hand, reduces the expected cost of producing q_C at a faster rate for the stronger bidder than his weaker opponent. Our results are opposed to the prevalent notion in the literature which suggests that smaller firms are more likely to bribe than their larger competitors (Shleifer and Vishny [2], Svensson [3]).

Our numerical simulations suggest that the first-score auction dominates the second whenever the stronger bidder is approached for a bribe in the latter format. In case the weaker bidder is favored, the buyer's preference is reversed when the probability of corruption is high (Result 3). The buyer becomes inclined to the second-score auction when the probability of corruption rises along with bidder asymmetry, as she correctly infers that the agent will approach the weaker bidder for a bribe but will be unable to manipulate bids in favor of the preferred bidder. Lastly, neither of the scoring auctions is ex-ante efficient.

2 Related Literature

This study provides non-trivial contributions to two strands of literature: procurement design using multi-attribute auctions and corruption. Che [4] is the first study to provide a theoretical understanding of multi-attribute auctions using quasi-linear scoring rules. It demonstrates that an optimal scoring rule is an anonymous one that "systematically discriminates against quality". Other significant contributions to the scoring auctions literature include those by Branco [5], Asker and Cantillon [6],[7], David et al. [8] and Nishimura [9]. Simple multidimensional generalizations in the form of first- and second-score auctions are shown to implement the optimal mechanism when seller's type spaces comprise fixed and marginal costs of binary nature (Asker and Cantillon[7])[2], when quality is multidimensional (Nishimura[9]), and when costs are correlated (Branco[5])[3]. David et al.[8] consider a weighted scoring rule similar to ours and show numerically that both first- and second-score auctions approximate the optimal mechanism. In our paper, because suppliers are no longer assumed to be symmetric, the optimal revelation mechanism recommends the use of a non-anonymous scoring method that handicaps the strong firm. In the likely event of this form of discrimination being considered unethical or illegal, implementing the optimal auction is then quite difficult. Therefore, we evaluate the two "better, faster, cheaper" [4] scoring auctions individually to see which one performs better for the buyer, with or without corruption.

The issue of corruption in auctions has been addressed very recently in the literature. It is generally assumed that the procurement agent can, in exchange for a bribe, either agree to readjust a bid (Compte et al.[11],Auriol[12], Burguet and Perry[13]) or can compromise on the quality of the good provided (Burguet and Che[14],Celentani and Ganuza[1], Burguet[15], Huang and Xia[16],). Our study falls into the second category and is inspired by Celentani and Ganuza[1], which studies the impact of competition on corruption in a first-score auction with symmetric bidders. Using a quasi-linear scoring rule, Celentani and Ganuza[1] show that under optimal auction, higher the probability of corruption, lesser should be the weight attached to quality.

Burguet and Che[14] on the other hand, consider a procurement auction in which two firms simultaneously submit multidimensional bids and offer bribes to the agent, to whom the administration of the auction has been delegated. In a complete information setting, they show that the auction is inefficient when the manipulation power of the agent is large and that the optimal scoring rule announced by the buyer is biased against quality. Huang and Xia[16] use a framework similar to that of Burguet and Che[14] and show that the buyer could ben-

[2]A complete characterization of equilibrium bidding behavior when sellers have multidimensional type spaces and submit multidimensional quality bids can be found in Asker and Cantillon[6].

[3]When costs are correlated, Branco[5] shows that either a first- or a second-score auction accompanied by bargaining over the level of quality to be provided implements the optimal mechanism.

[4]see Chen-Ritzo et al.[10]

efit from corruption[5]. Burguet[15] uses a contrasting approach as he characterizes quality corruption after the procurement contract has been awarded as the contracted supplier bribes the inspector to manipulate quality assessments in its favor. He shows that the optimal contracting rules hinge on the extent to which bribes are manipulable (fixed, variable, or uncertain) with downward distortions in quality for all cases.

3 The Model

Consider a government (or private) agency which has to procure an indivisible heterogeneous good that can be produced according to different specifications. For the sake of brevity of exposition, we denote the various possible specifications as quality level, q. The buyer can procure the good from one of two asymmetric firms, labeled strong (s) and weak (w). These firms submit two-dimensional bids, in which they specify the quality of the good that they will deliver and the payment that they expect in return (p). This is in contrast to a procurement auction involving a homogeneous good, in which bidders submit bids comprising prices only. In addition, we assume that the buyer is unable to verify the quality of good produced, and delegates the procurement task to an agent, who possesses more information about quality than the buyer. The agent is assumed to be risk-neutral and earns a wage which is normalized to zero.

The buyer (or principal) is assumed to be risk-neutral and her utility from a contract $(q, p) \in \mathbb{R}^2_+$ is increasing in q and decreasing in the second component. The buyer's utility is given by $U(q, p) = V(q) - p$, with $V'(q) > 0$, $V''(q) < 0$, $\lim_{q \to 0} V'(q) = \infty$ and $lim_{q\to\infty} V'(q) = 0$[6]. We assume that the buyer uses the functional form $V(q) = \log q$. Firms on the other hand, incur a cost of production, $c(q, \theta_i)$, where q is the delivered quality and θ_i is an efficiency parameter, which is private information. The cost function is assumed to be continuous and differentiable in both of its components such that $c_q > 0$, $c_{\theta_i} < 0$, $c_{qq} \geq 0$, $c_{\theta_i \theta_i} > 0$, and $c_{q\theta_i} < 0$ (Branco[5]). The efficiency parameters of the two firms are assumed to be independently distributed; while the efficiency parameter of

[5]Dastidar and Mukherjee[17] describe a different mode of corruption in public procurement than ours, where a politician in charge of a scoring auction decides the scoring rule while receiving a bribe from the winning firm.

[6]In addition to the assumption that the utility from quality is increasing at a diminishing rate, we make assumptions in order to ensure the existence of an interior solution.

the stronger firm, θ_s, is assumed to be uniformly distributed over $[\underline{\eta}, \eta_s]$, θ_w is assumed to be uniformly distributed over $[\underline{\eta}, \eta_w]$, with $\eta_w < \eta_s$ i.e.

$$F_s(\theta) = \frac{\theta - \underline{\eta}}{\eta_s - \underline{\eta}} \forall \theta \in [\underline{\eta}, \eta_s] \text{ and } F_w(\theta) = \frac{\theta - \underline{\eta}}{\eta_w - \underline{\eta}} \forall \theta \in [\underline{\eta}, \eta_w]$$

such that F_s (first-order) stochastically dominates F_w over $[\underline{\eta}, \eta_w]$. Both the firms are risk-neutral with a payoff function $U_i^F(q,p) = p - c(q, \theta_i)$, where p is payment received and q represents quality delivered in case the firm wins the contract.

In order to evaluate the two-dimensional bids, the buyer uses a scoring rule, which associates a score to any price-quality tuple that is submitted as a bid by a firm. The scoring rule used in our framework is a quasi-linear function $S(q,p) = s(q) - p$, for which $s(.)$ is increasing at least for $q \leq \arg\max_q s(q) - c(q, \theta_i)$ (Che[4])[7]. The buyer uses one of two mechanisms to evaluate the bids: first-score or second-score auction. In the first-score auction, the bidder with the highest score wins the auction, receives a transfer equal to the quoted price and delivers the good at the specified quality level. If the second-score auction is used, the firm with the highest score once again wins the auction, but is required to match the highest rejected score through any quality-price combination.

The procurement agent in charge of running the auction is assumed to be one of two types: honest or corrupt. We assume that the agent is corrupt with probability x and honest with the remaining probability. An honest agent runs the auction using a scoring rule pre-specified by the buyer and verifies the quality of the delivered good. In the case of the first-score auction, the delivered quality must be the same as the quality which was specified in the two-dimensional bid; in the second-score auction, the winning firm chooses a price and quality combination that generates the second-highest score – it produces the good at the quality level specified in its bid and is paid an amount which yields the highest rejected score (Che[4]). For expositional convenience, we assume that there is no moral hazard problem between an honest agent and the winning firm. A corrupt agent, on the other hand, approaches one of the two firms for a bribe before they submit their respective bids. In return, he promises to manipulate the bid of the approached firm and to declare it the winner, while allowing it to supply the good at a lower quality.

[7] The buyer is said to use a *Naive Scoring Rule* if it represents her true preferences, i.e. if $S(q,p) = U(q,p)$ or some monotone transformation of it.

While the buyer is aware that the agent is corrupt with some probability, she has to assign a positive weight to the quality component of the bid[8]. The use of such allocation rules creates a scope for corruption, as the agent uses his superior information to manipulate the bids in favor of the approached firm. To limit the distortionary powers of the agent, the buyer is assumed to use a scoring rule that 'under-rewards' quality. To account for this, we introduce a parameter λ in the scoring rule, which assigns a weight to the quality component of the bid. The scoring rule is then rewritten as $S(q,p;\lambda) = s(q;\lambda) - p$. We assume that due to legal constraints, the buyer uses the same discernment index for both the firms. To explicate how the presence of a corrupt agent modifies the way the auction is run, we present the extensive form of the game.

1. Nature selects the efficiency parameters θ_s, θ_w and the agent's type.

2. The agent privately learns his type while firms are privately informed of their efficiency parameters.

3. The principal publicly announces the scoring rule along with the auction format i.e. either the *first-score* or the *second-score* auction without observing the agent's type or the efficiency parameters.

4. The procurement agent acquires the good through the announced auction mechanism.

    A. If the agent is honest, the auction process goes about in the manner described above. The firm which submits the highest score is declared the winner[9] and its identity is revealed. The agent verifies that the delivered quality is indeed q and certifies it.

    B. (i) If the agent is corrupt, he chooses to approach one of the two firms for a bribe *before* the bids are placed. Once the agent meets a firm, he learns its efficiency parameter and demands a bribe. In exchange, the agent promises to manipulate bids in order to declare that firm as the winner (under conditions to be explained later) and to permit the it to produce a lower quality level, q_C.

    (ii) The firm approached for a bribe either accepts or rejects the offer.

[8]Several studies have shown that the optimal allocation mechanism of a heterogenous good must assign values to both quality and price. See Che [4], Branco [5] and Laffont and Tirole [18].

[9]Ties are broken using a random draw.

(a) If the firm accepts the offer, it is declared the winner under the pretext that it offers the best price and quality combination under the pre-specified scoring rule. It then receives a payment (which is specified below), produces the good at a quality level q_C which is lower than the one specified in the bid, and pays out the bribe to the agent, all not necessarily in the same order. The agent then verifies that the quality delivered is q_C and certifies it as the one specified in the bid, q. We assume that the corrupt arrangement is detected with a probability γ, in which case, both the agent and the firm are penalized with penalties P^A and P^B respectively.

(b) If the firm rejects the offer, the agent does not get another chance to contact the other firm. The procurement process is managed honestly as described earlier. Irrespective of whether the firm accepts the agent's offer or rejects it, the agent announces the identity of the winning firm, the payment to be made and the quality supplied.

While the payoff of the honest agent is normalized to zero, the payoff of the (risk-neutral) corrupt agent is equal to the expected bribe minus γP^A. As an extension, we also discuss the case of an agent who is not only corrupt but also vengeful. The timing of events of the game with such an agent is identical to the one described above, with the exception that in case the bribe offer is rejected, the agent tries to use his manipulation power to ensure that the firm he contacted for a bribe never wins the auction.

3.1 Discussion of the Model

While the timing of events in our model is similar to that of Celentani and Ganuza[1], there are some crucial differences. For example, we assume that the probability with which the agent is corrupt is exogenously given. This is in contrast to the framework used by Celentani and Ganuza[1], where the agent's decision to be corrupt is endogenously determined. In their model, the agent decides whether or not to incur the cost of corruption before he is matched with a firm. However, it must be the case that after the agent is matched with a firm and learns its type, it remains profitable for the agent to engage in a corrupt transaction with that firm. Otherwise, the agent would choose not to ask for a bribe even though β is sunk. Celentani and Ganuza[1] assumes that this condition holds for certain parameter values (Assumption 2) and proceeds to derive the probability of corruption. We avoid making any such assumption in our model and assume that the agent is corrupt with some exogenously given probability which assists in isolating the effect of bidders' asymmetry on the agent's choice.

The second important distinction between the two settings is that rather than being randomly matched with one of the two firms, the agent chooses which firm to approach for a bribe. This follows from the *raison d'être* of our paper, which studies how a corrupt agent makes decisions in an asymmetric bidder setting. The revelation of private information however, is common across the two setups. The agent decides whom to approach without knowing the efficiency parameter of either firm, gets to know the type of the firm he approaches and makes his proposal. The agent's type on the other hand is revealed only to the approached firm, who updates $x = 1$. In case a firm is not contacted before it submits its bid, the firm (along with the buyer) continues to believe that the agent is corrupt with probability x.

It is noteworthy to point out the various ways in which quality features in different parts our model. For example, the buyer's utility, as well as the score obtained from a bid, depend on q. However, the quality used in the score function is as certified by the agent and need not be the true quality of the good. The buyer's utility on the other hand depends on the latter, which may never be perfectly revealed to the buyer. The agent and the two firms, on the other hand, are perfectly able to distinguish between the various quality levels. This implies that the minimum quality q_C is observable to both the agent and the approached firm. The buyer is however aware that in case the agent is corrupt and is successful in manipulating bids, the supplied quality will be q_C.

Here, we would also like to elucidate the information that is revealed by the agent to the buyer at the end of auction. We do so as the information provided has a strong link with the extent to which the score of the favored firm can be manipulated by the corrupt agent. We assume that the agent only announces the identity of the winner, the transfer to be made to the winning firm and the quality that the winner is supposed to supply, to the buyer. The buyer then calculates the score of the winning firm in the first-score auction and the second highest score in case of the second-score auction. This prevents the corrupt agent from manipulating bids in favor of the weaker firm in the second-score auction when $\theta_s \in [\eta_w, \eta_s]$ (see section 5.2). Monitoring by the buyer also puts bounds on the extent to which bids can be manipulated and renders the average payment made in the corrupt mechanism to be equal to the one in the honest counterpart, thereby making it harder for the buyer to detect such corrupt arrangements.

The probability of detection, γ, and the penalties P^A, P^B are in no way essential to the results of our model. They do not determine the identity of the firm who is approached by the corrupt agent. They however, do play a role in determining whether or not the agent decides to contact one of the two firms for a bribe in the first place. By making bribes inadmissible, the penalties provide one of the means to prevent corruption in this setup.

4 Auction without Corruption

4.1 First-Score Auction

When the agent is honest (or the agent is corrupt but the approached firm turns down the bribe offer), the auction process is similar to the one without delegation. This setup constitutes the benchmark model for our study. In the first-score auction, the winner is the firm with the price-quality combination that generates the highest score. The buyer remunerates the winner with a transfer equal to the price that was bid and in return is supplied with a quality, q. The objective function of a firm is given by

$$Max_{q,p}\pi(q,p|\theta_i) = \{p - c(q,\theta_i)\}Pr[win|S(q,p)] \tag{1}$$

For the firms to make non-negative profits, it should be such that $p \geq c(q,\theta_i)$. In other words, $S(q,p;\lambda) = s(q;\lambda) - p \leq s(q;\lambda) - c(q,\theta_i) \leq \max_q s(q;\lambda) - c(q,\theta_i)$. We define $S_o(\theta_i;\lambda) = \max_q s(q;\lambda) - c(q,\theta_i)$ and $q_o(\theta_i;\lambda) = argmax_q s(q;\lambda) - c(q,\theta_i)$ for all future purposes.

Lemma 1. *The promised quality level chosen by a firm of type θ_i is*

$$q_o(\theta_i;\lambda) = argmax_q s(q;\lambda) - c(q,\theta_i)$$

for all $\theta_i \in (\underline{\eta}, \eta_i]$; $i = s, w$.

Proof. This is immediate from Lemma 1 Che[4]. The asymmetry amongst the bidders has no effect on the firms' quality bids. □

The maximum score that any firm with type θ_i can offer, $S_o(\theta_i;\lambda)$, is termed as the *productive potential* of that firm by Che[4]. Using the Envelope Theorem, $S_o'(\theta_i;\lambda) = -c_{\theta_i}(q,\theta_i) > 0$ which implies that $S_o(.)$ is strictly increasing and that its inverse exists. As a result, this $S_o(\theta_i;\lambda)$ can be treated as the **pseudo-valuation** of the contract by the firm with type θ_i and enables us to transform the two-dimensional procurement auction problem into a unidimensional one.

Let $v_i = S_o(\theta_i;\lambda)$ and follows a cumulative distribution function say, $\mathscr{H}_i(.)$ with density function $h_i(.)$, where

$$\mathscr{H}_i(S) = Pr[S_o(\theta_i;\lambda) \leq S] = F_i(S_o^{-1}(S;\lambda))$$

Also, let $b_i = S(q_o(\theta_i;\lambda), p)$. Then $v_i - b_i = p - c(q_o(\theta_i;\lambda), \theta_i)$. The firm's problem can then be written as

$$Max_{b_i}\ \pi(b_i, v_i) = (v_i - b_i)Pr[win|b_i] \tag{2}$$

Suppose that in equilibrium, the strong and the weak firm follow strategies $\beta_s(S_o(\theta_s;\lambda))$ and $\beta_w(S_o(\theta_w;\lambda))$ respectively. Further, let us assume that these strategy functions are increasing and differentiable. Let their inverse functions be $\phi_s \equiv \beta_s^{-1}$ and $\phi_w \equiv \beta_w^{-1}$. The least efficient firm will bid nothing but its own pseudo-valuation i.e. $\beta_i(S_o(\underline{\eta};\lambda)) = S_o(\underline{\eta};\lambda)\ \forall i = s, w$. Also, it can be seen that the bid offered by the strong firm of type η_s would be equal to that offered by the weak firm of type η_w.

Let $\bar{b} \equiv \beta_s(S_o(\eta_s;\lambda)) \equiv \beta_w(S_o(\eta_w;\lambda))$. The expected profit of the firm i with pseudo-valuation $v_i \equiv S_o(\theta_i;\lambda)$ who bids $b < \bar{b}$, given that firm j bids using $\beta_j(.)$ is

$$\pi(b, v_i) = (v_i - b)Pr[b > \beta_j(S_o(\theta_j;\lambda))] = (v_i - b)\mathscr{H}_j(\phi_j(b)) \tag{3}$$

The first-order condition for firm i is

$$(v_i - b)h_j(\phi_j(b))\phi_j'(b) - \mathscr{H}_j(\phi_j(b)) = 0$$

Since $\phi_i(b) = v_i$, the first-order conditions when both the firms maximize their expected profits simultaneously are,

$$(\phi_i(b) - b)h_j(\phi_j(b))\phi_j'(b) - \mathscr{H}_j(\phi_j(b)) = 0\ \ \forall i, j = s, w \tag{4}$$

Substituting for $\mathscr{H}(.)$ and $h(.)$ in equation (3), we get

$$(\phi_i(b) - b)f_j(S_o^{-1}(\phi_j(b);\lambda))(S_o^{-1}(\phi_j(b);\lambda))' = F_j(S_o^{-1}(\phi_j(b);\lambda))\ \ \forall i, j = s, w \tag{5}$$

A solution to this system of differential equations with relevant boundary conditions constitutes an equilibrium of the first-score auction. It is difficult to obtain a general solution to the system of differential equations due to the unspecified functional form of the pseudo-valuation, $S_o(\theta_i;\lambda)$. We therefore consider an example where $S_o(\theta_i;\lambda)$ takes a specific form.

Let $s(q) = 2\sqrt{\lambda q}$ and $c(q;\theta_i) = \frac{q}{\theta_i}$, where λ is the weight assigned to quality and represents the *discernment index* of the buyer[10] [11]. Higher the λ, less

[10]For the functional form $c(q;\theta_i) = \frac{q}{\theta_i}$, the inverse of the efficiency parameter of a firm may be interpreted as its marginal cost.

[11]The use of $s(q) = \sqrt{\lambda}\cdot 2\sqrt{q}$ instead of $s(q) = \lambda \cdot 2\sqrt{q}$ makes no difference in our comparisons as for $\lambda \in [0,1]$, $f : \lambda \longrightarrow \sqrt{\lambda}$ is essentially a monotonic, one-to-one correspondence. Rather,

discerning the buyer is in terms of perceiving and controlling the manipulative power of the procurement agent. Using the above functional forms, we get $S_o(\theta_i;\lambda) = \lambda\theta_i$ and $q_o(\theta_i) = \lambda\theta_i^2$. The cumulative distribution functions associated with strong and weak firm types are

$$\mathscr{H}_i(S) = Pr[S_o(\theta_i) \leq S] = Pr\left[\theta_i \leq \frac{S}{\lambda}\right] = F_i\left(\frac{S}{\lambda}\right) \text{ for } i = s, w.$$

Substituting for $\mathscr{H}_j(.)$ and $h_j(.)$ in equation (4), we get the first-order conditions

$$\begin{aligned}
(\phi_i(b) - b) f_j\left(\frac{\phi_j(b)}{\lambda}\right)\left(\frac{\phi_j'(b)}{\lambda}\right) &= F_j\left(\frac{\phi_j(b)}{\lambda}\right) \qquad \forall\ i, j = s, w \\
\implies (\phi_i(b) - b)\frac{1}{\eta_j - \underline{\eta}}\frac{\phi_j'(b)}{\lambda} &= \frac{\phi_j'(b)/\lambda - \underline{\eta}}{\eta_j - \underline{\eta}} \\
\implies (\phi_i(b) - b)\phi_j'(b) &= \phi_j(b) - \lambda\underline{\eta} \qquad (6)
\end{aligned}$$

Let $\lambda\underline{\eta} = \underline{\lambda}$ and $\lambda\eta_i = \lambda_i\ \forall i = s, w$ such that the pseudo-valuations of firms lie in the intervals $[\underline{\lambda}, \lambda_w]$ and $[\underline{\lambda}, \lambda_s]$. This system of differential equations with boundary conditions $\phi_i(\underline{\lambda}) = \underline{\lambda}\ \forall i = s, w$ is solvable for ϕ_i. These, upon inverting, yield bidding strategies as described in Proposition 1.

Proposition 1. *1. The bidding strategies of the firms are*

$$\beta_j^{FS}(\lambda\theta) = \lambda\underline{\eta} + \frac{1}{k_j(\lambda\theta - \lambda\underline{\eta})}\left(-1 + \sqrt{1 + k_j(\lambda\theta - \lambda\underline{\eta})^2}\right) \qquad \forall j = s, w$$

2. In this first-score auction, each firm in equilibrium offers

$$q_s^{FS}(\theta;\lambda) = q_w^{FS}(\theta;\lambda) = q_o(\theta;\lambda) = \lambda\theta^2$$

$$p_i^{FS}(\theta;\lambda) = \lambda(2\theta - \underline{\eta}) - \frac{1}{k_i\lambda(\theta - \underline{\eta})}\left(-1 + \sqrt{1 + k_i\lambda^2(\theta - \underline{\eta})^2}\right)$$

where $k_j = \dfrac{1}{\lambda^2}\left(\dfrac{1}{(\eta_i - \underline{\eta})^2} - \dfrac{1}{(\eta_j - \underline{\eta})^2}\right) \qquad \forall\ i, j = s, w.$

Proof. See Appendix. □

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the former functional form is useful in getting tractable equilibrium bidding functions.

Corollary 1. *The expected winning offer in this first-score auction is*

$$E(q^{FS}(\lambda)) = \sum_{i \neq j}^{s,w} \int_{\underline{\eta}}^{\eta_i} q_i^{FS}(\theta;\lambda) \frac{\phi_j^{FS}(\beta_i^{FS}(\lambda\theta))/\lambda - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta$$

$$E(p^{FS}(\lambda)) = \sum_{i \neq j}^{s,w} \int_{\underline{\eta}}^{\eta_i} p_i^{FS}(\theta;\lambda) \frac{\phi_j^{FS}(\beta_i^{FS}(\lambda\theta))/\lambda - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta$$

Proof. Immediate. □

4.2 Second-Score Auction

In the second-score auction, the firm with the highest score wins the contract but is now asked to match the second highest score in the auction. However, it is not essential for the winning firm to offer the exact price and quality combination of that of the losing firm. Again, we define $S_o(\theta;\lambda) = \max_q s(q;\lambda) - c(q,\theta)$ and $q_o(\theta;\lambda) = \text{argmax}_q s(q;\lambda) - c(q,\theta)$ and convert the two-dimensional auction into a unidimensional one (Che[4]). As in the standard second-price auction, it is a weakly dominant strategy for the firms to bid their own valuations (or pseudo-valuations). Therefore, each firm would bid a score equal to $S_o(\theta_i;\lambda)$. A direct consequence of such bidding is that the most efficient firm wins the auction.

Proposition 2. *1. The bidding strategies of the firms in the second-score auction are*

$$\beta_i^{SS}(\theta) = S_o(\theta;\lambda) \qquad \forall i = s,w$$

2. In the second-score auction, each firm in equilibrium offers

$$q_i^{SS}(\theta;\lambda) = q_o(\theta;\lambda)$$
$$p_i^{SS}(\theta;\lambda) = c(q_o(\theta;\lambda),\theta) \qquad \forall i = s,w$$

Corollary 2. *The expected winning offer in the second-score auction is*

$$E(q)^{SS} = E\{q_o(\theta_1;\lambda)\}$$
$$E(p)^{SS} = E\{s(q_o(\theta_1;\lambda)) - s(q_o(\theta_2;\lambda)) + c(q_o(\theta_2;\lambda),\theta_2)\}$$

where $\theta_1 = Max(\theta_s,\theta_w)$ *and* $\theta_2 = Min(\theta_s,\theta_w)$.

4.3 Optimal Auction

Before we analyze the decisions of the corrupt agent across the two auction formats and the preferred scoring rule of the buyer, we solve for the optimal mechanism when the agent is honest. An honest agent manages the mechanism as per the buyer's preferences and allocates the project accordingly. The proof of the following proposition closely resembles that of Proposition 1 in Celentani and Ganuza [1].

Proposition 3. *The optimal mechanism with an honest agent allocates the good to the firm with the highest* $\frac{\theta_i^2}{\eta_i}$ *and requires it to supply* $q_i = \frac{\theta_i^2}{\eta_i}$.

Proof. See Appendix A. □

The interpretation of this proposition is similar to that of the optimal mechanism as provided by Myerson [19]. In his seminal work on auctions with independent private valuations, Myerson [19] shows that the revenue-maximizing mechanism allocates the good to the buyer with the highest (positive) *virtual valuation*. In a symmetric setting, this translates to allocating the object to the buyer with highest valuation, provided it exceeds a certain minimum level. The optimal mechanism, in that case, can be implemented through a variety of standard procedures, such as a first- or second-price sealed bid auction, with an appropriately chosen reserve price (Riley and Samuelson [20]). Che [4] extended results from Myerson [19] to multidimensional auctions where simple generalizations of unidimensional formats in the form of first- and second-score auctions were shown to implement the optimal mechanism. In the asymmetric setting, however, the optimal mechanism requires detailed information regarding the distributions from which the buyers' valuations are drawn and therefore, cannot be implemented by a simple auction format. Several studies, for this reason, have chosen to focus on the revenue ranking of commonly used auctions, such as first- and second-price auctions (Kirkegaard [21], Maskin and Riley [22], Mares and Swinkels [23], Mares and Swinkels [24]). In our setting, the optimal mechanism dictates that the object be given to the supplier with highest $\frac{\theta_i^2}{\eta_i}$, in contrast to the symmetric bidder case, where it would be allocated to the seller with highest θ_i. The optimal mechanism, therefore, penalizes firms with higher maximum valuations. From Propositions (1), (2) and (3) it is apparent that neither the first-score nor the second-score auction implements the optimal mechanism with a symmetric scoring rule.[12]

[12] In case of a symmetric auction, such that $\eta_w = \eta_s = \overline{\eta}$, the optimal mechanism would

5 Auction with Corruption

5.1 First-Score Auction

We analyze the decisions of a corrupt agent in the first-score auction using backward induction. After the buyer announces the procurement mechanism, the corrupt agent chooses to approach one of the two firms, learns its private information and makes a take-it-or-leave-it offer. As part of the offer, the agent promises to manipulate the bids of the approached firm and declare it the winner with probability one. He also offers to allow the approached firm to produce the good at an exogenously given minimum quality, q_C, which is lower than the one in the "concocted" bid. In return he asks for a bribe. In what follows, we first explain the bid manipulation process and later, solve the bidder selection problem of the corrupt agent.

5.1.1 Score Manipulation

Let firm i be the one who is approached by the corrupt agent and θ_j be the type of his rival. Once the bid $\beta_j(\lambda\theta_j)$ is submitted, the agent manipulates the bid of the favored firm. He does this by making the bid of the favored firm appear to be the bid of a hypothetical firm of type $\theta' = \theta_j + \psi_j(\theta_j)$ such that $\psi_j(.)$ satisfies

$$\psi_j(\theta_j) \geq 0 \;\; \forall \theta_j \in [\underline{\eta}, \eta_j] \tag{7}$$

$$\theta_j + \psi_j(\theta_j) \leq \eta_j \;\; \forall \theta_j \in [\underline{\eta}, \eta_j] \tag{8}$$

$$\text{and} \int_{\underline{\eta}}^{\eta_j} \frac{p_i^{FS}(\lambda^{-1}\phi_i(\beta_j(\lambda\theta')))}{(\eta_j - \underline{\eta})} d\theta_j = E(p)^{FS} \tag{9}$$

The interpretation of these conditions is the same as in Celentani and Ganuza [1]. The third condition is based on the argument that the buyer might initiate investigations following an announcement of an apriori low probability offer. To subvert this contingency, the corrupt agent ensures that the average payment made in the corrupt arrangement is equal to the one managed by an honest agent. This implies that the firm of type θ_i is announced to be of (higher) type

allocate the good to the supplier with the highest θ_i and require him to supply $q_i = (1-x)\theta_i^2$ in the case where the agent is corrupt with probability x and $V(q) = \overline{\eta}\log q$. The first-score auction with scoring rule $S(q,p;\lambda) = 2\sqrt{\lambda q} - p$ then implements the optimal mechanism with $\lambda = 1 - x$.

θ_i^a such that

$$\beta_i(\lambda\theta_i^a) = \beta_j(\lambda\theta')$$
$$\Rightarrow \theta_i^a = \lambda^{-1}\beta_i^{-1}(\beta_j(\lambda\theta')) = \lambda^{-1}\phi_i(\beta_j(\lambda\theta')).$$

Therefore, the ex-post payment made to firm i is $p_i^{FS}(\lambda^{-1}\phi_i(\beta_j(\lambda\theta')))$, while the quantity it should have supplied is $\lambda(\theta_i^a)^2$. However, at the time when the corrupt deal is struck, the type of the rival firm is unknown to both the agent and the favored firm. In order to satisfy condition (9), we must have

$$\int_{\underline{\eta}}^{\eta_j} \frac{p_i^{FS}(\lambda^{-1}\phi_i(\beta_j(\lambda\theta')))}{(\eta_j - \underline{\eta})} d\theta_j = E(p)^{FS} = \sum_{i \neq j}^{s,w} \int_{\underline{\eta}}^{\eta_i} p_i^{FS}(\theta;\lambda) \frac{\phi_j^{FS}(\beta_i^{FS}(\lambda\theta))/\lambda - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta. \tag{10}$$

Since $p(\theta_i) = 2\lambda\theta_i - \beta_i(\lambda\theta_i)$ in our example and $\beta_i(\theta_i^a) = \beta_j(\theta')$, we can also write

$$\int_{\underline{\eta}}^{\eta_j} \frac{2\phi_i(\beta_j(\lambda\theta')) - \beta_j(\lambda\theta')}{(\eta_j - \underline{\eta})} d\theta_j = E(p)^{FS}. \tag{11}$$

5.1.2 Bribe Setting and Bidder Selection

Now that we have delineated the payment that the favored firm receives in the corrupt arrangement, we can solve for the maximum bribe that it would be willing to pay and the firm that the agent would in fact favor in the first-score auction. One important question to ask is whether the bidding functions are going to be different from what they are when the agent is honest given that the agent is corrupt with some probability. The answer is in the negative. Since the bidding takes place *after* the agent and the chosen firm have colluded, the firm that was not made the offer can in no way alter its bidding behavior so as to increase the probability of it being declared the winner. As for the favored firm, it can decide to bid in whichever manner as the agent has guaranteed its win even before the bidding process has taken place. One such way would be to adhere to bidding the same way as in the honest mechanism.

The expected payoff of firm i when it chooses to accept the agent's offer to bribe, denoted by π_i^C, is given as

$$\pi_i^C = E(p)^{FS} - B_i - c(q_C, \theta_i) - \gamma P^B. \tag{12}$$

Firm i chooses to accept the bribe offer when the expected profits earned are at least as high as the expected profits earned by the firm when it decides to turn down the offer. That is to say, $\pi_i^C \geq \pi_i^H$, where π_i^H denotes expected profit in the honest mechanism. In equilibrium, the bribe paid by the firm i is

$$B_i = E(p)^{FS} - \pi_i^H - c(q_C, \theta_i) - \gamma P^B,\ i = s, w. \tag{13}$$

The expected payoff to the agent when the firm rejects his offer is zero. Since the agent is unaware of the type of either firm (at the time of making his decision), the expected payoff to the agent from approaching firm i is

$$\begin{aligned} E_{\theta_i}(\pi_i^A) &= E_{\theta_i}(B_i - \gamma P^A) \\ &= E(p)^{FS} - E_{\theta_i}(\pi_i^H + c(q_C, \theta_i)) - \gamma(P^A + P^B) \end{aligned} \tag{14}$$

The agent asks the weaker firm for a bribe when $E_{\theta_w}(\pi_w^A) \geq E_{\theta_s}(\pi_s^A) > 0$ and approaches the stronger firm when $E_{\theta_s}(\pi_s^A) > E_{\theta_w}(\pi_w^A) > 0$. These conditions lead us to the following proposition.

Proposition 4. *In the first-score auction with asymmetric bidders, the corrupt agent chooses to approach the weaker (stronger) firm when*

$$E_{\theta_s}(\pi_s^H + c(q_C, \theta_s)) \geq (<) E_{\theta_w}(\pi_w^H + c(q_C, \theta_w))$$

and $Max_i\{E(B_i) - \gamma P^A\} > 0$.

For $s(q) = 2\sqrt{\lambda q}$ and $c(q, \theta) = \dfrac{q}{\theta}$, the condition in Proposition 4 becomes

$$E_\theta(\pi_s^H) + q_C E_\theta(\frac{1}{\theta_s}) \geq E_\theta(\pi_w^H) + q_C E_\theta(\frac{1}{\theta_w}) \tag{15}$$

where

$$E_\theta(\pi_i^H) = \int_{\underline{\eta}}^{\eta_i} (\lambda\theta - \beta_i(\lambda\theta)) \frac{\phi_j^{FS}(\beta_i^{FS}(\lambda\theta))/\lambda - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta$$

and

$$E_\theta\left(\frac{1}{\theta_i}\right) = \frac{1}{(\eta_i - \underline{\eta})} \log\left(\frac{\eta_i}{\underline{\eta}}\right)\ \forall i = s, w.$$

Using Taylor's expansion it is easy to show that for $\eta_w < \eta_s$, the function $f(x) = \dfrac{1}{(x - \underline{\eta})} \log\left(\dfrac{x}{\underline{\eta}}\right)$ is always decreasing in x. So, for an exogenously given q_C, it is always true that $E_\theta\left(\dfrac{1}{\theta_s}\right) \leq E_\theta\left(\dfrac{1}{\theta_w}\right)$. However, it is not straightforward to compare expected profits under the honest case. We have,

$$E_\theta(\pi_i^H) = \frac{1}{k_i\lambda(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} \int_{\underline{\eta}}^{\eta_i} \left(-1 + \sqrt{1 + k_i(\lambda\theta - \lambda\underline{\eta})^2}\right) d\theta\ \forall i = s, w \tag{16}$$

which when simplified gives us,

$$E_\theta(\pi_s^H) = \frac{1}{2\lambda^2 k_s^{3/2}(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})}\Bigg[\lambda\sqrt{k_s}(\eta_s - \underline{\eta})\left(-2 + \sqrt{1 + k_s(\lambda\eta_s - \lambda\underline{\eta})^2}\right) + \log\left(\lambda\sqrt{k_s}(\eta_s - \underline{\eta}) + \sqrt{1 + k_s(\lambda\eta_s - \lambda\underline{\eta})^2}\right)\Bigg]$$

$$E_\theta(\pi_w^H) = \frac{1}{2\lambda^2 k_s^{3/2}(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})}\Bigg[\lambda\sqrt{k_s}(\eta_w - \underline{\eta})\left(2 - \sqrt{1 - k_s(\lambda\eta_w - \lambda\underline{\eta})^2}\right) - \sin^{-1}\left(\lambda\sqrt{k_s}(\eta_w - \underline{\eta})\right)\Bigg].$$

When $\eta_s = \eta_w = \eta$, we get $E_\theta(\pi_s^H) = E_\theta(\pi_w^H) = \dfrac{\lambda}{6(\eta - \underline{\eta})}[2\eta^2 - (\underline{\eta}+1)\eta - (\underline{\eta}^2 + 2\underline{\eta})]$.

To analyze the condition in Proposition 4 further, we consider a particular example. We assume that $\eta_w = \underline{\eta} + \frac{1}{1+\alpha}$ and $\eta_s = \underline{\eta} + \frac{1}{1-\alpha}$, such that $\alpha \in (0,1)$ denotes the level of asymmetry between the two firms[13]. For $\alpha = 0, \eta_s = \eta_w = \underline{\eta} + 1$, such that the firms are symmetric and the expected profits for both the firms are equal to $\frac{\lambda}{6}$. We then define a variable $Y^{FS}(.)$ such that

$$Y^{FS}(\alpha, \lambda; q_C, \underline{\eta}) = E_\theta(\pi_s^H) + q_C E_\theta\left(\frac{1}{\theta_s}\right) - E_\theta(\pi_w^H) - q_C E_\theta\left(\frac{1}{\theta_w}\right). \tag{17}$$

Following Proposition 4, the agent will approach the stronger (weaker) firm for a bribe if $Y^{FS}(\alpha, \lambda; q_C) < 0 (\geq 0)$.

Place figure 2 here.

We find from figure 2 that the agent will approach the weaker firm for a bribe for higher values of α and that this region shrinks as $\underline{\eta}$ becomes higher[14]. Typically for each value of $\underline{\eta}$, we get a cutoff $\widehat{\alpha}_{FS}(\underline{\eta})$ such that for $\alpha < \widehat{\alpha}_{FS}(\underline{\eta})$, the strong firm wins the contract for all values of λ[15]. We draw the level curves for Y^{FS} in figure 2 under the assumption that $q_C = \lambda\underline{\eta}^2$. This ensures that the minimum quality allowed under the corrupt transaction generates (cost) savings for

[13] With $\eta_w = \underline{\eta} + \frac{1}{1+\alpha}, \eta_s = \underline{\eta} + \frac{1}{1-\alpha}$ and $\alpha \in (0,1)$ it can be shown that $\frac{\partial E(p)^{FS}}{\partial \underline{\eta}} = -\lambda < 0$.

[14] In these panels, the zero level curve gives us a visual representation of locus of combinations of α and λ which gives $Y^{FS}(\alpha, \lambda; q_C) = 0$. For any combination of α and λ to the right of this level curve, the agent prefers to approach the weaker firm.

[15] For example, if $\underline{\eta} = 30, \widehat{\alpha}_{FS} = 0.733$, for $\underline{\eta} = 65, \widehat{\alpha}_{FS} = 0.833$ and if $\underline{\eta} = 100$, $\widehat{\alpha}_{FS} = 0.865$.

firms of all efficiencies (except the lowest). Also, when the probability of detection is small, our simulations suggest that at any given production efficiency level, $Max\{E(B_w)-\gamma P^A, E(B_s)-\gamma P^A\} > 0\ \forall \alpha, \lambda$.

Result 1. *If* $\eta_w = \underline{\eta} + \frac{1}{1+\alpha}$, $\eta_s = \underline{\eta} + \frac{1}{1-\alpha}$ *and* $q_C = \lambda \underline{\eta}^2$, $\exists\ \widehat{\alpha}_{FS}(\underline{\eta}) \in (0,1)$ *such that for* $\alpha < \widehat{\alpha}_{FS}(\underline{\eta})$ *the agent approaches the stronger firm for a bribe in the first-score auction. The cutoff* $\widehat{\alpha}_{FS}$ *is rising in* $\underline{\eta}$.

The intuition behind the above result is as follows. The (expected) payment obtained by the supplier through the corrupt arrangement is the same as under no corruption. The maximum bribe that the firm is willing to pay to the agent, in that case, is falling in the (expected) cost of corruption. This includes (i) the opportunity cost of accepting the fraudulent deal, π_i^H and (ii) the cost of producing the minimum quality level, q_C. Thus, higher the outside option, or lower the productive efficiency of the firm, lower will be the bribe that the agent can extract from the supplier.

The stronger firm has an inherent advantage in terms of the second component, as its (expected) cost of producing q_C is lower than that of its weaker counterpart. For the limiting case of α close to zero, firms have access to outside options of the same value. This drives the agent to the stronger firm for lower values of bidder asymmetry. However, as firms become more asymmetric, the expected payoff of the stronger firm from turning down the bribe offer rises faster than its weaker counterpart. This latter effect dominates the cost advantage enjoyed by the stronger firm to the extent that for higher values of α, the agent approaches the weaker firm for a bribe. The comparative statics analysis for $\underline{\eta}$ is easier. The lower bound of the efficiency parameter has no effect on the outside option of either firm; however, an increase in $\underline{\eta}$ leads to a smaller reduction in the expected cost of producing q_C for the weaker firm and results in the agent choosing the stronger firm for higher values of $\underline{\eta}$.

While Result 1 helps us analyze the decisions of a corrupt agent for particular combinations of parameter values, it fails to provide a holistic perspective as to how frequently the corrupt agent approaches the weaker firm over his stronger counterpart. In order to address this shortcoming, we use a bootstrapped simulation technique as follows.

(1) We first create a sample of 100 observations of Y^{FS} by using randomly generated values of $\alpha, \lambda, \underline{\eta}$ and q_C. For consistency, we take only those values of q_C such that $0 < q_C \leq \lambda \underline{\eta}^2$.

(2) We then calculate the sample proportion for which $Y^{FS} \geq 0$ and label the same by $\widehat{p}_i$.

(3) We iteratively repeat steps 1 and 2 10,000 times. This process generates a distribution of sample proportions for which $Y^{FS} > 0$, as shown in figure 3.
In our case, the population parameter of interest is the proportion of values for which $Y^{FS} \geq 0$, p. From each sample, we get a $\widehat{p}_i$, which is a random variable. Following the Central Limit Theorem, this statistic is known to approximately follow a Normal distribution with mean p and standard deviation $\sqrt{\frac{p(1-p)}{100}}$. We find that $p = 0.0792$, $\sqrt{\frac{p(1-p)}{100}} = 0.0269$ and that there is no sample which has less than 81% of observations with $Y^{FS} < 0$.

Place figure 3 here.

5.2 Second-Score Auction

In the second-score auction, the expected payoff of a firm from accepting the agent's offer (denoted by π_i^C above) can no longer be calculated using the procedure as in the first-score auction. In fact, we show that the manipulation powers of the agent are curtailed to some extent in the second-score auction. Specifically, we argue that *the agent cannot declare the weak (favored) supplier as the winner whenever* $\theta_s \in [\eta_w, \eta_s]$. In that case, the highest rejected score[16] would be $S_o(\theta_s;\lambda) = \lambda\theta_s \geq \lambda\eta_w$. With $S_o(\theta;\lambda)$ being an increasing function of θ, the type of the winning (weaker) firm would then have to be higher than η_w, which is impossible. Similarly, we argue that there is no need for the agent to manipulate bids if the favored firm is strong and $\theta_s \in [\eta_w, \eta_s]$.

5.2.1 *When the agent asks the weaker firm for a bribe.*

While making an offer to the weaker firm, the agent asks for a bribe B_w with the stipulation that he will manipulate bids only if the rival's type $\theta_s < \eta_w$. In this case, bid manipulation takes place through a process similar to that in the first-score auction. The favored firm θ_w is shown to be a hypothetical firm with type $\theta' = \theta_s + \psi_s(\theta_s)$ and is allowed to supply quality q_C. The weaker firm wins the auction and is asked to provide a price-quality tuple that generates the highest rejected score

$$S(q(\theta_s), p(\theta_s)) = s(q_o(\theta_s)) - c(q_o(\theta_s), \theta_s).$$

[16]As discussed in section 3.1, the highest rejected score is revealed to the buyer in the second-score auction.

For a firm to profess to supply quality $q_o(\theta')$, the payment requested should be such that

$$s(q_o(\theta')) - p = s(q_o(\theta_s)) - c(q_o(\theta_s), \theta_s)$$
$$\Rightarrow p \equiv p(\theta_s) = s(q_o(\theta')) - s(q_o(\theta_s)) + c(q_o(\theta_s), \theta_s).$$

The payoff to the weaker firm from accepting the corrupt arrangement will be

$$\pi_w^C = \Pr(\theta_s < \eta_w)\left[E_{\theta_s}(p(\theta_s)|\theta_s < \eta_w) - c(q_C, \theta_w)\right] - B_w - \gamma P^B.$$

In order to ensure that the bribe offer is accepted, the agent makes a take-it-or-leave-it offer to the weaker firm, such that the expected payoff from accepting the bribe offer $\pi_w^C \geq \pi_w^H(\theta_w)$. The agent therefore sets

$$B_w = \Pr(\theta_s < \eta_w)\left[E_{\theta_s}(p(\theta_s)|\theta_s < \eta_w) - c(q_C, \theta_w)\right] - \gamma P^B - \pi_w^H(\theta_w).$$

and earns an expected payoff $E(\pi_w^A)$ from approaching the weaker supplier, where

$$E(\pi_w^A) = \Pr(\theta_s < \eta_w) E_{\theta_w}\left[E_{\theta_s}(p(\theta_s)|\theta_s < \eta_w)\right] \\ - E_{\theta_w}\left[\Pr(\theta_s < \eta_w) c(q_C, \theta_w)\right] - E_{\theta_w}(\pi_w^H(\theta_w)) - \gamma(P^B + P^A). \quad (18)$$

However, in case $\theta_s \geq \eta_w$, the agent is unable to manipulate bids in favor of θ_w and will have to make a payment to the stronger firm. This payment, $p'(\theta_w, \theta_s)$ is given by

$$p'(\theta_w, \theta_s) = s(q_o(\theta_s)) - s(q_o(\theta_w)) + c(q_o(\theta_w), \theta_w)$$

such that the expected payment made by the corrupt agent is given by $E_{\theta_w}\left[E_{\theta_s}(p'(\theta_w, \theta_s)|\theta_s \geq \eta_w)\right]$.

It is important to point out here that in order to avoid detection of the corrupt arrangement, the agent should not set the expected payment made to the favored firm equal to the expected payment made by an honest agent. Instead, while manipulating bids, he should ensure that the total expected payment made by him is equal to the total expected payment made by his honest self.[17] The manipulation function, ψ_s, must therefore satisfy the following conditions:

$$\psi_s(\theta_s) \geq 0 \;\; \forall \theta_s \in [\underline{\eta}, \eta_w) \quad (19a)$$
$$\theta_s + \psi_s(\theta_s) \leq \eta_w \;\; \forall \theta_s \in [\underline{\eta}, \eta_w) \quad (19b)$$
$$\Pr(\theta_s < \eta_w) E_{\theta_w}\left[E_{\theta_s}(p(\theta_s)|\theta_s < \eta_w)\right] + \Pr(\theta_s \geq \eta_w) E_{\theta_w}\left[E_{\theta_s}(p'(\theta_w, \theta_s)|\theta_s \geq \eta_w)\right] = E(p)^{SS}. \quad (19c)$$

[17] In the first-score auction on the other hand, as well as in the symmetric setting of Celentani and Ganuza [1], expected payment made by the corrupt agent to the favored firm is equal to the total expected payment made by the corrupt agent. This in turn, is set equal to the total expected payment made in the honest mechanism.

5.2.2 ***When the agent asks the stronger firm for a bribe.***

In order to ensure that the strong firm wins the contract, the agent manipulates the score only when $\theta_s \in [\underline{\eta}, \eta_w)$. Alternatively, if $\theta_s \geq \eta_w$, the stronger firm can be declared as the winner without any manipulation. In this case, in exchange for a bribe, the agent allows the favored firm to supply the minimum quality q_C. Let $P(.), p'(.)$ be the payment functions that are used to determine the payment made when $\theta_s < \eta_w$ and $\theta_s \geq \eta_w$ respectively.

(i) If $\theta_s \in [\underline{\eta}, \eta_w)$, the agent manipulates bids as described above, by assigning a hypothetical type θ'' to the stronger firm such that $\theta'' = \theta_w + \psi_w(\theta_w)$. The stronger firm wins and is asked to provide a quality-price tuple that generates the highest rejected score

$$S(q(\theta_w), p(\theta_w)) = s(q_o(\theta_w)) - c(q_o(\theta_w), \theta_w).$$

The quality supplied is shown to be $q_o(\theta'')$, and the payment requested ensures

$$\begin{aligned} s(q_o(\theta'')) - P &= s(q_o(\theta_w)) - c(q_o(\theta_w), \theta_w) \\ \Rightarrow P \equiv P(\theta_w) &= s(q_o(\theta'')) - s(q_o(\theta_w)) + c(q_o(\theta_w), \theta_w). \end{aligned}$$

The expected bribe conditional on $\theta_s < \eta_w$ will therefore be

$$\begin{aligned} E_{\theta_s}(B_s|\theta_s < \eta_w) = E_{\theta_s}\left[E_{\theta_w}(P(\theta_w))|\theta_s < \eta_w\right] - E_{\theta_s}\left[c(q_C, \theta_s)|\theta_s < \eta_w\right] \\ - E_{\theta_s}[\pi_s^H(\theta_s)|\theta_s < \eta_w] - \gamma P^B. \end{aligned} \tag{20}$$

(ii) On the other hand, if θ_s lies in $[\eta_w, \eta_s]$, there is no need for bid manipulation as the stronger firm's type (and bid) will be higher than its weaker counterpart. The winner then requests a payment which is concomitant with the second-highest score

$$p' \equiv p'(\theta_w, \theta_s) = s(q_o(\theta_s)) - s(q_o(\theta_w)) + c(q_o(\theta_w), \theta_w).$$

The expected bribe for $\theta_s \geq \eta_w$ is therefore,

$$\begin{aligned} E_{\theta_s}(B_s'|\theta_s \geq \eta_w) = E_{\theta_s}\left[E_{\theta_w}(p'(\theta_w, \theta_s))|\theta_s \geq \eta_w\right] - E_{\theta_s}\left[c(q_C, \theta_s)|\theta_s \geq \eta_w\right] \\ - E_{\theta_s}[\pi_s^H(\theta_s)|\theta_s \geq \eta_w] - \gamma P^B. \end{aligned} \tag{21}$$

while the expected payoff to the corrupt agent from approaching the stronger firm is given by[18]

$$E(\pi_s^A) = pr(\theta_s < \eta_w)E_{\theta_s}(B_s|\theta_s < \eta_w) + \Pr(\theta_s \geq \eta_w)E_{\theta_s}(B_s'|\theta_s \geq \eta_w) - \gamma P^A.$$

[18]This is due to the fact that the agent first finds out the type of the approached firm and then asks for a bribe. The type of the rival firm remains unobserved.

Using (20) and (21), we get

$$E(\pi_s^A) = \Pr(\theta_s < \eta_w) E_{\theta_s}\left[E_{\theta_w}(P(\theta_w)) | \theta_s < \eta_w\right] + \Pr(\theta_s \geq \eta_w)$$
$$E_{\theta_s}\left[E_{\theta_w}(p'(\theta_w, \theta_s)) | \theta_s \geq \eta_w\right] - E_{\theta_s}(c(q_C, \theta_s)) - E_{\theta_s}(\pi_s^H(\theta_s)) - \gamma(P^A + P^B). \quad (22)$$

Finally, in order to avoid detection, the manipulation function $\psi_w(.)$ must satisfy conditions similar to those specified by (19a-19c) above,

$$\psi_w(\theta_w) \geq 0 \;\; \forall \theta_w \in [\underline{\eta}, \eta_w] \quad (23a)$$

$$\theta_w + \psi_w(\theta_w) \leq \eta_w \;\; \forall \theta_w \in [\underline{\eta}, \eta_w] \quad (23b)$$

$$\Pr(\theta_s < \eta_w) E_{\theta_s}\left[E_{\theta_w}(P(\theta_w)) | \theta_s < \eta_w\right] + \Pr(\theta_s \geq \eta_w) E_{\theta_s}\left[E_{\theta_w}(p'(\theta_w, \theta_s)) | \theta_s \geq \eta_w\right] = E(p)^{SS} \quad (23c)$$

Comparing the conditions (19c) and (23c), we find that the manipulation functions $\psi_w(.)$ and $\psi_s(.)$ must be constructed in a way which ensures

$$E_{\theta_w}\left[E_{\theta_s}(p(\theta_s) | \theta_s < \eta_w)\right] = E_{\theta_s}\left[E_{\theta_w}(P(\theta_w)) | \theta_s < \eta_w\right]. \quad (24)$$

This can be easily done since with $\theta_s < \eta_w$, we are in a situation identical to that of a symmetric auction. The scores can therefore be manipulated using the same functions, as in Celentani and Ganuza [1].

Using the payoffs in equations (18) and (22), we can now solve for the agent's decision. If $E(\pi_w^A) \geq E(\pi_s^A) > 0$, the agent will approach the weaker firm for a bribe. That is to say if,

$$\Pr(\theta_s < \eta_w) E_{\theta_w}\left[E_{\theta_s}(p(\theta_s) | \theta_s < \eta_w)\right] - E_{\theta_w}\left[\Pr(\theta_s < \eta_w) c(q_C, \theta_w)\right] - E_{\theta_w}(\pi_w^H(\theta_w))$$
$$-\gamma(P^B + P^A) \geq \Pr(\theta_s < \eta_w) E_{\theta_s}\left[E_{\theta_w}(P(\theta_w)) | \theta_s < \eta_w\right]$$
$$+\Pr(\theta_s \geq \eta_w) E_{\theta_s}\left[E_{\theta_w}(p'(\theta_w, \theta_s)) | \theta_s \geq \eta_w\right] - E_{\theta_s}(c(q_C, \theta_s)) - E_{\theta_s}(\pi_s^H(\theta_s)) - \gamma(P^A + P^B)$$

then the agent will contact the weaker supplier. Using equation (24) we simplify the above condition and get

$$E_{\theta_s}(c(q_C, \theta_s)) - E_{\theta_w}\left[\Pr(\theta_s < \eta_w) c(q_C, \theta_w)\right] + E_{\theta_s}(\pi_s^H(\theta_s)) - E_{\theta_w}(\pi_w^H(\theta_w)) \geq$$
$$\Pr(\theta_s \geq \eta_w) E_{\theta_s}\left[E_{\theta_w}(p'(\theta_w, \theta_s)) | \theta_s \geq \eta_w\right]$$
$$\Rightarrow \frac{q_C}{\eta_s - \underline{\eta}} \ln\left(\frac{\eta_s}{\underline{\eta}}\right) - \frac{q_C}{\eta_w - \underline{\eta}} \ln\left(\frac{\eta_w}{\underline{\eta}}\right) \frac{\eta_w - \underline{\eta}}{\eta_s - \underline{\eta}} + E_{\theta_s}(\pi_s^H(\theta_s)) - E_{\theta_w}(\pi_w^H(\theta_w)) \geq$$
$$\Pr(\theta_s \geq \eta_w) E_{\theta_s}\left[E_{\theta_w}(p'(\theta_w, \theta_s)) | \theta_s \geq \eta_w\right]$$
$$\Rightarrow \frac{q_C}{\eta_s - \underline{\eta}} \ln\left(\frac{\eta_s}{\eta_w}\right) + E_{\theta_s}(\pi_s^H(\theta_s)) - E_{\theta_w}(\pi_w^H(\theta_w)) \geq \Pr(\theta_s \geq \eta_w) E_{\theta_s}\left[E_{\theta_w}(p'(\theta_w, \theta_s)) | \theta_s \geq \eta_w\right]. \quad (25)$$

Profit to firm i under the honest mechanism is

$$\pi_i^H(\theta_i) = E\left[(S_o(\theta_i) - b) \cdot I_{\beta_i > b}\right]$$

with firms bidding their pseudo-valuations i.e. $\beta_i = S_o(\theta_i) = \lambda\theta_i$. The expected profit $E(\pi_i^H(\theta_i))$ can be shown to be equal to $\frac{\lambda(\eta_i-\underline{\eta})^2}{6(\eta_j-\underline{\eta})}$ $\forall i,j = s,w$, while the right-hand side of the above inequality simplifies to $\int\limits_{\eta_w}^{\eta_s}\int\limits_{\underline{\eta}}^{\eta_w}(2\lambda\theta_s-\lambda\theta_w)\frac{1}{(\eta_s-\underline{\eta})(\eta_w-\underline{\eta})}d\theta_w d\theta_s$. The condition under which the corrupt agent will approach the weaker firm is given in the following proposition.

Proposition 5. *In the second-score auction with asymmetric bidders, the corrupt agent chooses to approach the weaker firm when*

$$Y^{SS}(\alpha,\lambda;q_C,\underline{\eta}) = \frac{q_C}{\eta_s-\underline{\eta}}\ln\left(\frac{\eta_s}{\eta_w}\right) + \frac{\lambda(\eta_s-\underline{\eta})^2}{6(\eta_w-\underline{\eta})} - \frac{\lambda(\eta_w-\underline{\eta})^2}{6(\eta_s-\underline{\eta})}$$
$$-\int\limits_{\eta_w}^{\eta_s}\int\limits_{\underline{\eta}}^{\eta_w}(2\lambda\theta_s-\lambda\theta_w)\frac{1}{(\eta_s-\underline{\eta})(\eta_w-\underline{\eta})}d\theta_w d\theta_s \geq 0$$

and to approach the stronger firm otherwise.

We substitute $\eta_s = \underline{\eta}+\frac{1}{1-\alpha}$ and $\eta_w = \underline{\eta}+\frac{1}{1+\alpha}$ after simplifying the above condition to get,

$$Y^{SS} = q_C(1-\alpha)\ln\left[\frac{\underline{\eta}+\frac{1}{1-\alpha}}{\underline{\eta}+\frac{1}{1+\alpha}}\right] + \frac{2\lambda\alpha}{3(1-\alpha^2)^2}(3\alpha-3\underline{\eta}+2\alpha^2+3\alpha\underline{\eta}+3\alpha^2\underline{\eta}$$
$$-3\alpha^3\underline{\eta}-3). \tag{26}$$

In order to better understand when the agent approaches the weaker firm for a bribe, we set $q_C = \lambda\underline{\eta}^2$ and plot Y^{SS} against α for different values of $\underline{\eta}$ (see figure 3). We find that $Y^{SS} \geq 0$ for $\alpha \geq \widehat{\alpha}_{SS}(\underline{\eta})$ and that the cutoff value $\widehat{\alpha}_{SS}$, is rising in $\underline{\eta}$. The intuition behind this is as follows. The expected payment to the stronger firm when $\theta_s \geq \eta_w$, as well as the expected profit under the honest mechanism, $E_{\theta_s}(\pi_s^H(\theta_s))$, rise with α. While a higher payment leads to a larger bribe demand, a higher outside option for the stronger firm on the other hand, reduces the bribe that can be extorted from the same firm. It is easily verifiable that $E_{\theta_s}(\pi_s^H(\theta_s)) \to \infty$ as $\alpha \to 1$, such that the latter effect dominates the former. This leads the agent to approach the weaker firm for higher levels of bidder asymmetry. We also find that both the terms in the expression (26) are decreasing in $\underline{\eta}$, such that as $\underline{\eta}$ rises, it is unlikely that the agent will approach the weaker firm for a bribe.

Place figure 3 here.

In order to corroborate our analytical results we ran the following simulations. In the first simulation we used a bootstrapped procedure to get an estimate of the probability with which a corrupt agent approaches the weaker firm.
(1) We first created a sample of 100 observations of Y^{SS} by using randomly generated values of $\alpha, \lambda, \underline{\eta}$ and q_C. We assumed $\alpha, \lambda \sim U[0,1]$, drew $\underline{\eta}$ from a uniform distribution $[0, \overline{H}]$ and allowed $q_C \sim U[0, \lambda\underline{\eta}^2]$. [19]
(2) In the second step, we calculated the proportion of sample observations for which $Y^{SS} > 0$, $\widehat{p}$.
(3) Finally, we iteratively repeated the first two steps 10,000 times.
This bootstrapped procedure generated a distribution of sample proportions for $Y^{SS} > 0$. In this case, the population parameter of interest is the proportion of values with $Y^{SS} > 0, p$. From each sample we get a $\widehat{p}_i$, which is known to approximately follow a Normal distribution with mean p and standard deviation $\sqrt{\frac{p(1-p)}{n}}$. The mean and standard deviation for each distribution, along with their associated confidence interval estimates are reported in panel A of the following table for various upper bounds of $\underline{\eta}$. The corresponding distributions of sample proportions with such parametric restrictions are shown in figure 4 in Appendix B.

Insert table (1) here.

We ran a similar simulation for the minimum value of α at which $Y^{SS} > 0$, represented by $\widehat{\alpha}_{SS}$. The procedure followed was identical to the one described above, except that we used specific values for $\underline{\eta}$ and in step 2, for each sample, we calculated $\widehat{\alpha}_{SS,i}$. The results from this simulation are presented in panel B of table (1), in which we report the mean and standard deviation of the distributions of $\widehat{\alpha}_{SS,i}$ for different values of $\underline{\eta}$. We find that the distribution of $\widehat{\alpha}_{SS,i}$ approximately follows a Normal distribution for $\underline{\eta} = 10, 50$ and a (left-skewed) Beta distribution for $\underline{\eta} = 500, 1000$ and $10,000$, as is apparent from figure 5 in Appendix B. Further evidence in favor of our second analytical result is provided by a simulation with parametric restrictions identical to the one we used for $\widehat{\alpha}_{SS}$, in which we plotted Y^{SS} against α, λ and q_C. We find that as $\underline{\eta}$ rises, $Y^{SS} > 0$ for higher values of asymmetry (see figure 6 in Appendix B).

Result 2. *In the second-score auction (i) the frequency with which the agent approaches the weaker firm decreases as $\underline{\eta}$ increases and (ii) the agent approaches the weaker firm whenever $\alpha > \widehat{\alpha}_{SS}(q_C, \underline{\eta}) \in (0,1)$, where $\frac{\partial \widehat{\alpha}_{SS}}{\partial \underline{\eta}} > 0$.*

[19] H took values 10, 50, 500, 1000 and 10,000.

The bid manipulation process described in sections 5.1.2 and 5.2 involved the corrupt agent rigging the bid of the favored firm, by (mis)representing its type to be slightly higher than that of the opponent. Such falsification fails to work for the weaker firm in the second-score auction when $\theta_s > \eta_w$, as the highest losing score has to be shown to be higher than $\lambda\eta_w$. Bid manipulation, in this case, leads both the buyer and the losing firm to update their belief to $x = 1$. We believe that constraining the agent to announce the identity of the winner further obstructs manipulation of this form and others, as it raises the probability of detection. It should be noted that in the symmetric and asymmetric first-score auction, it makes no difference whether or not the identity of the winner is revealed, as the agent can manipulate bids $\forall \theta_w, \theta_s$.

6 When the agent is Corrupt and Vengeful

As an extension of our model we assume that the procurement agent in addition to being corrupt, is *vengeful*, i.e. if the agent approaches either of the firms for a bribe and the firm refuses, the agent uses his manipulation power to ensure that the firm which turned down his offer does not win the auction afterwards. We analyze corruption in the first-score and second-score auctions in this modified setting.

6.1 First-Score Auction

When the agent is corrupt and vengeful, the maximum bribe that a firm is willing to pay to the agent is higher than that when the agent is corrupt. This is due to the vengeful nature of the agent who ensures that the value of the outside option to the firm, $\pi_i^H = 0$. The approached firm therefore agrees to pay a bribe B_i as long as $E(p)^{FS} - c(q_C, \theta_i) - B_i - \gamma P^B \geq 0$. The equilibrium bribe will be $B_i = E(p)^{FS} - c(q_C, \theta_i) - \gamma P^B$, such that the agent asks the strong firm to bribe if $E_{\theta_s}(\pi_s^A) = E(B_s - \gamma P^A) > E(B_w - \gamma P^A) > 0$. We find that this is indeed the case since $E(1/\theta_s) < E(1/\theta_w)$. Thus, in the first-score auction, the corrupt and vengeful agent will approach the stronger firm irrespective of the level of asymmetry and discernment index.

6.2 Second-Score Auction

We argued in section 5.2 above, that the agent can manipulate bids only when $\theta_s \in [\underline{\eta}, \eta_w)$. This argument continues to hold when the agent is corrupt and vengeful. On the other hand, the stronger firm wins the contract for sure if its type $\theta_s \geq \eta_w$. In that case, the agent asks the firm to pay a bribe B'_s in lieu of which the agent certifies quality q_C as $q_o(\theta_s)$.

1. *The agent asks the stronger firm for a bribe.*

   Again, we assume that (P, B) and (p', B') are the payment and bribe pair for the strong firm when $\theta_s \in [\underline{\eta}, \eta_w)$ and $\theta_s \in [\eta_w, \eta_s]$ respectively. For θ_s realized in $[\underline{\eta}, \eta_w)$, the bribe will be $B_s = E_{\theta_w}(P(\theta_w)) - c(q_C, \theta_s) - \gamma P^B$, where $P(\theta_w) = s(q_o(\theta_w + \psi_w(\theta_w))) - s(q_o(\theta_w)) + c(q_o(\theta_w), \theta_w)$. This is the bribe at which the firm will be indifferent between accepting the bribe offer and rejecting it. The expected bribe in this case will be

   $$E(B_s|\theta_s < \eta_w) = E_{\theta_s}[E_{\theta_w}(P(\theta_w))|\theta_s < \eta_w] - E_{\theta_s}[c(q_C, \theta_s)|\theta_s < \eta_w] - \gamma P^B.$$

   Similarly, the expected bribe for $\theta_s \geq \eta_w$ is given by

   $$\begin{aligned} E(B'_s|\theta_s \geq \eta_w) = E_{\theta_s}[E_{\theta_w}(p'(\theta_w, \theta_s))|\theta_s \geq \eta_w] - E_{\theta_s}[c(q_C, \theta_s)|\theta_s \geq \eta_w) \\ - E_{\theta_s}[\pi_s^H(\theta_s)|\theta_s \geq \eta_w] - \gamma P^B \end{aligned}$$

   since the agent will not be able to prevent the stronger firm from winning the auction in case the bribe offer is turned down. In order to avoid detection, the agent uses a manipulation function $\psi_w(.)$ which satisfies conditions identical to the ones specified in section 5.2. Combining these cases, the expected payoff to the agent from approaching the stronger firm is

   $$\begin{aligned} E(\pi_s^A) &= \Pr(\theta_s < \eta_w)E(B_s|\theta_s < \eta_w) + \Pr(\theta_s \geq \eta_w)E(B'_s|\theta_s \geq \eta_w) - \gamma P^A \\ &= \Pr(\theta_s < \eta_w)E_{\theta_s}[E_{\theta_w}(P(\theta_w))|\theta_s < \eta_w] + \Pr(\theta_s \geq \eta_w)E_{\theta_s}[E_{\theta_w}(p'(\theta_w, \theta_s))|\theta_s \geq \eta_w] \\ &\quad - E_{\theta_s}[c(q_C, \theta_s)] - \Pr(\theta_s \geq \eta_w)E_{\theta_s}[\pi_s^H(\theta_s)|\theta_s \geq \eta_w] - \gamma(P^A + P^B). \end{aligned} \tag{27}$$

2. *The agent asks the weaker firm for a bribe.*

   In this case, the expected payoff of the weaker firm is

   $$\pi_w^C = P(\theta_s < \eta_w)E_{\theta_w}[(E_{\theta_s}(p(\theta_s)|\theta_s < \eta_w) - c(q_C, \theta_w)] - B_w - \gamma P^B$$

   as the weak firm is portrayed to be of type $\theta' = \theta_s + \psi_s(\theta_s)$, provided $\theta_s < \eta_w$. While the payment made in this case is $p(\theta_s) = s(q_o(\theta')) - s(q_o(\theta_s)) +$

$c(q_o(\theta_s),\theta_s)$, the function $\psi_s(.)$ satisfies conditions (19a)-(19c). The weak firm can either accept to bribe and earn π_w^C or alternatively, get punished by the agent and lose the contract, thereby earning nothing. The bribe demanded will be

$$B_w = P(\theta_s < \eta_w)E_{\theta_w}[E_{\theta_s}(p|\theta_s < \eta_w) - c(q_C,\theta_w)] - \gamma P^B$$

such that the following expected payoff accrues to the agent

$$E(\pi_w^A) = P(\theta_s < \eta_w)E_{\theta_w}[E_{\theta_s}(p(\theta_s)|\theta_s < \eta_w) - c(q_C,\theta_w)] - \gamma(P^A + P^B). \quad (28)$$

Using equations (27), (28) and (24), we can now determine whom the agent will approach for a bribe. We represent the difference in expected payoffs by $Y_v^{SS} = E(\pi_w^A) - E(\pi_s^A)$ and summarize the required condition as follows.

Proposition 6. *In the second-score auction with a corrupt and vengeful agent, the agent chooses to approach the weaker firm for a bribe whenever $Y_v^{SS} \geq 0$, i.e.*

$$\frac{q_C}{\eta_s - \underline{\eta}} \ln\left(\frac{\eta_s}{\eta_w}\right) + \int_{\eta_w}^{\eta_s} \pi_s^H(\theta_s)\frac{1}{\eta_s - \underline{\eta}}d\theta_s \geq \int_{\eta_w}^{\eta_s}\int_{\underline{\eta}}^{\eta_w}(2\lambda\theta_s - \lambda\theta_w)\frac{1}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})}d\theta_w d\theta_s.$$

Since

$$\frac{1}{\eta_s - \underline{\eta}}\int_{\eta_w}^{\eta_s}\pi_s^H(\theta_s)d\theta_s = \frac{\lambda(\eta_s - \underline{\eta})^2}{6(\eta_w - \underline{\eta})} - \frac{\lambda(\eta_w - \underline{\eta})^2}{6(\eta_s - \underline{\eta})},$$

the condition in Proposition 6 is the same as in Proposition 5. All the results from section 5.2, therefore, go through in this setting.

7 Preferred Scoring Auction of the Buyer

Before we analyze the preferred scoring rule of the buyer, we discuss the optimal mechanism with an agent who is corrupt with probability x. Under the assumption $V(q_i) = \log q_i$, we have already shown that the optimal mechanism without delegation recommends that the contract be allocated to the firm with highest $\frac{\theta_i^2}{\eta_i}$ and that the winner should supply quality $q_i = \frac{\theta_i^2}{\eta_i}$. In the optimal mechanism with corruption, we assume that the agent approaches one of the two firms for a bribe; in exchange (where possible), he declares the firm as the one who should be awarded the contract as per the optimal allocation rule and allows the favored supplier to produce quality q_C. We assume that as the agent manipulates bids, he ensures that the (total) expected payment made is equal to the (total) expected payment made by an honest agent, in order to avoid detection of corruption.

Proposition 7. *The optimal mechanism with an agent who is corrupt with probability x, allocates the good to the supplier with the highest $\frac{\theta_i^2}{\eta_i}$. (a) If the agent approaches the stronger firm for a bribe, the optimal mechanism recommends that the supplied quality should be $q_i^O = (1-x)\frac{\theta_i^2}{\eta_i}$. (b) If the agent approaches the weaker firm and declares the weaker firm as the winner, the supplied quality should be $q_w^O = (1-x)\frac{\theta_w^2}{\eta_w}$. If the stronger firm is declared as the winner, the optimal quality should be $q_s^O = \frac{\theta_s^2}{\eta_s}$.*

Proof. See Appendix A. □

The intuition behind setting $q_s^O = \frac{\theta_s^2}{\eta_s}$ when the stronger firm is announced as the winner even when the weaker firm is approached for a bribe, is that the buyer correctly infers that the agent was unable to manipulate bids in favor of the weaker firm. This happens only when the announced type of the stronger firm satisfies $\frac{\theta_s^2}{\eta_s} \geq \frac{\eta_w^2}{\eta_w} = \eta_w$. The agent then manages the auction as if there was no corruption. The implementation of the optimal mechanism, as in the case without corruption, demands identity-based discrimination and requires additional information regarding the upper bounds of the distributions of bidders' valuations. Since $q_i = \lambda\theta_i^2$ in both the first-score and second-score auction, neither scoring rule implements the optimal mechanism. We assume that due to legal constraints, the buyer uses a symmetric, anonymous scoring rule in which she assigns $\lambda = 1-x$ and compares expected utility between the first-score and second-score auction.

The expected utility to the buyer from the first-score auction is given by

$$\begin{aligned} EU^{FSC} &= (1-x)[E(V(q^{FS})) - E(p)^{FS}] + x[V(q_C) - E(p)^{FS}] \\ &= (1-x)E(V(q^{FS})) - E(p)^{FS} + xV(q_C) \end{aligned}$$
$$= \frac{1}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} \sum_i^{s,w} \int_{\underline{\eta}}^{\eta_i} \left[(1-x)\log(\lambda\theta_i^2) - p_i(\theta_i)\right] \frac{(\theta_i - \underline{\eta})}{\sqrt{1 + k_i\lambda^2(\theta_i - \underline{\eta})^2}} d\theta_i + x\log q_C \tag{29}$$

where $p_i(\theta_i) = \lambda(2\theta_i - \underline{\eta}) - \frac{1}{k_i\lambda(\theta_i - \underline{\eta})}\left(-1 + \sqrt{1 + k_i\lambda^2(\theta_i - \underline{\eta})^2}\right)$ and $k_i = -k_j = \frac{1}{\lambda^2}\left(\frac{1}{(\eta_j - \underline{\eta})^2} - \frac{1}{(\eta_i - \underline{\eta})^2}\right) \forall i, j = s, w$. The expected utility from the second-score auction, however, depends on the firm the corrupt agent approaches for a bribe. If the agent approaches the stronger firm, the expected utility derived is

$$\begin{aligned} EU_s^{SSC} &= (1-x)[E(V(q^{SS})) - E(p)^{SS}] + x[V(q_C) - E(p)^{SS}] \\ &= (1-x)E(V(q^{SS})) + x\log q_C - E(p)^{SS} \end{aligned} \tag{30}$$

where[20],

$$\begin{gathered} E[V(q^{SS})] = E[\log(\lambda\theta_1^2)] = \log\lambda + E(\log\theta_1^2) \\ = \log\lambda + \int_{\underline{\eta}}^{\eta_w} 2\log x \frac{2(x-\underline{\eta})}{(\eta_w - \underline{\eta})(\eta_s - \underline{\eta})} dx + \int_{\eta_w}^{\eta_s} 2\log x \frac{1}{\eta_s - \underline{\eta}} dx \end{gathered} \tag{31}$$

and[21]

$$\begin{gathered} E(p)^{SS} = E[s(q_o(\theta_1)) - s(q_o(\theta_2)) + c(q_o(\theta_2), \theta_2)] = \lambda[2E(\theta_1) - E(\theta_2)] \\ = \lambda\left[2\frac{1}{6(\eta_s - \underline{\eta})}(\eta_w^2 - 2\underline{\eta}\eta_w - 2\underline{\eta}^2 + 3\eta_s^2) - \frac{1}{6(\eta_s - \underline{\eta})}\left[(3\eta_s - \eta_w)(\eta_w + \underline{\eta}) - 4\underline{\eta}^2\right]\right]. \end{gathered} \tag{32}$$

If the agent decides to approach the weaker firm instead, the corresponding expected utility is

$$\begin{aligned} EU_w^{SSC} = &(1-x)E(V(q^{SS})) + x\Pr(\theta_s \geq \eta_w)E[V(q_o(\theta_s))|\theta_s \geq \eta_w] \\ &+ x\Pr(\theta_s < \eta_w)\log q_C - E(p)^{SS} \end{aligned} \tag{33}$$

where $E(V(q^{SS}))$ and $E(p)^{SS}$ are given by (31) and (32) respectively.

In Section 5.2 we showed that the corrupt agent approaches the weaker firm for a bribe whenever $\alpha \geq \widehat{\alpha}_{SS}(q_C, \underline{\eta})$ and that the cutoff $\widehat{\alpha}_{SS}$ increases with

[20] $\theta_1 = Max\{\theta_w, \theta_s\}$ with the associated c.d.f.

$$G_1(x) = \begin{cases} \dfrac{(x-\underline{\eta})^2}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} & \text{if } x \in [\underline{\eta}, \eta_w] \\ \dfrac{x - \underline{\eta}}{\eta_s - \underline{\eta}} & \text{otherwise} \end{cases}.$$

[21] $\theta_2 = \min\{\theta_w, \theta_s\}$ with the associated c.d.f.

$$G_2(x) = \frac{(x-\underline{\eta})}{(\eta_s - \underline{\eta})} + \frac{(x-\underline{\eta})}{(\eta_w - \underline{\eta})} - \frac{(x-\underline{\eta})^2}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})}, x \in [\underline{\eta}, \eta_w].$$

$\underline{\eta}$. We find that $\widehat{\alpha}_{SS} \to 0.686$ as $\underline{\eta} \to 0$, such that for $\alpha \leq 0.68$, the agent always approaches the stronger firm for a bribe. The difference in expected utilities from the first- and second-score auction in that case is given by $Y^{US} = EU^{FSC} - EU_s^{SSC}$, where EU^{FSC} and EU_s^{SSC} are represented by equations (29) and (30) respectively. If $\alpha > 0.68$ on the other hand, we need to check for the favored firm of the agent; if the agent chooses the stronger firm, the difference in expected utilities continues to be Y^{US}, while $Y^{UW} = EU^{FSC} - EU_w^{SSC}$ represents the corresponding difference when the agent selects the weaker supplier.

(i) Agent approaches the stronger firm for a bribe. In order to simplify the analysis of the preferred scoring rule of the buyer, we assume that $\eta_s = \underline{\eta} + \frac{1}{1-\alpha}, \eta_w = \underline{\eta} + \frac{1}{1+\alpha}$ with $\alpha \in (0,1)$ and $q_C = \lambda \underline{\eta}^2$. We find that the *buyer prefers the first-score auction* over the second whenever the stronger firm is approached by the corrupt agent. Results from a bootstrapped simulation, similar to the one used in section 5.2 lend additional support in favor of our finding. In the first step of this simulation, we generated 100 observations of Y^{US} under the assumption that $\alpha \sim U[0, 0.68], x \sim U[0,1], q_C \sim U[0, \lambda \underline{\eta}^2]$ and $\underline{\eta} \sim U[0, H]$ with $H = 10, 50, 500$ and 1000. In the second step we calculated the proportion of observations for which $Y^{US} > 0$ ($\widehat{p}_s$), and then iteratively repeated the first two steps 10,000 times. The simulation yielded a degenerate distribution at 1 in all the cases.

(ii) Agent approaches the weaker firm for a bribe. If $Y^{SS} > 0$ we find that the buyer *prefers the second-score auction over the first, for lower values of* λ and that the proportion of observations for which $Y^{UW} < 0$ becomes smaller as $\underline{\eta}$ rises. The contour plots[22] of Y^{UW} against α and λ for different values of $\underline{\eta}$ (figure 7) provide a visual representation of our result. We ran a bootstrapped simulation similar to the one we used in case (i) above, with the only difference being $\alpha \sim U[0.68, 1]$. At the end of this procedure, we recorded the proportion of observations for which $Y^{SS} > 0$ ($\widehat{p}_w$) and the proportion of observations for which the first-score is preferred over the second, given that $Y^{SS} > 0$ ($\widehat{p}_{f|w}$). Results from this simulation procedure conform to our finding and are presented in table (2).

Result 3. *(i) The buyer prefers the first-score auction over the second whenever the stronger firm is approached for a bribe in the second-score auction. (ii) In case the weaker firm is favored, the buyer prefers the second-score auction when*

[22] These plots were drawn under the simplifying assumption that $\eta_s = \underline{\eta} + \frac{1}{1-\alpha}, \eta_w = \underline{\eta} + \frac{1}{1+\alpha}$ and $q_C = \lambda \underline{\eta}^2$.

$\lambda < \widehat{\lambda}(\alpha, \underline{\eta})$. However, the incidence with which the second-score is preferred becomes lower as $\underline{\eta}$ rises (i.e $\frac{\partial \widehat{\lambda}}{\partial \underline{\eta}} < 0$). (iii) The expected utility comparison results with the corrupt agent are identical to the one with a corrupt and vengeful agent.

The buyer's predilection for the second-score auction for higher levels of α and x can be explained as follows. First, the buyer correctly infers that the agent will approach the weaker firm for a bribe for higher values of α. As the probability of corruption rises, the buyer attributes a higher probability to being supplied with quality q_C rather than the one which is certified by the agent in the first-score auction. However, the scope of bid manipulation is lower in the second-score auction than in the first, as the agent manipulates bids only when $\theta_s < \eta_w$. The difference in expected quality between the second-score and the first-score auction in that case, is given by

$$Y^{EQ} = x \int_{\eta_w}^{\eta_s} \log(\lambda \theta_s^2) \frac{1}{\eta_s - \underline{\eta}} d\theta_s + x \frac{\eta_s - \eta_w}{\eta_s - \underline{\eta}} \log q_C - x \log q_C.$$

We find that Y^{EQ} is increasing in α but falling in λ and that the rate of change is decreasing in $\underline{\eta}$. Since $E(p)^{FS}, E(p)^{SS}$ along with $E(V(q^{FS}))$, $E(V(q^{SS}))$ vary with α, λ in a virtually identical manner, we posit that the behavior of Y^{EQ} alone is sufficient to explain the change in preference of the buyer (see figure 8).

As a robustness check, we repeat the analysis of the preferred scoring rule of the buyer under the assumptions $\lambda = 1 - x^2$, $\lambda = (1-x)^2$ and find that the results are similar to the ones described above for almost all parameter values. For $\lambda = 1 - x^2$, we get results analogous to the ones in result 3, except when the agent prefers to approach the weaker firm; the second-score auction is then seen to dominate the first-score auction provided $\underline{\eta} = 1$. In this case, the buyer prefers the second score auction as she assigns a larger weight to the quality component of the bid and Y^{EQ} attains its highest possible value. If $\lambda = (1-x)^2$ on the other hand, the results are once again similar to the case with $\lambda = 1 - x$, other than when $Y^{SS} < 0$; the buyer in that case, prefers the second-score auction over the first for low values of $\underline{\eta}$ and $\lambda \to 0$.

8 Discussion and Conclusion

Using a setup similar to that of Celentani and Ganuza [1], we first solved for the decision problem of a corrupt agent who uses information asymmetry regarding quality to his advantage and manipulates bids to favor a firm in exchange for a bribe. We then addressed the related question of the preferred scoring rule of the buyer, who is constrained to run the auction through a privately informed agent. While Celentani and Ganuza [1] show that corruption and competition might concomitantly increase under certain market conditions, our analysis provides insights into the extent to which a corrupt agent can manipulate bids in an asymmetric bidder setting and provides the resultant expected utility ranking across the standard scoring auctions.

We show, inter alia, that the corrupt agent approaches the stronger firm for a bribe in the first- and second-score auction for most parameter values and prefers the weaker firm when the level of asymmetry is sufficiently high. The incidence with which the agent proposes the weaker firm, however, becomes smaller as the minimum level of efficiency rises. Our results are driven largely by the outside options available to the bidders and the cost of providing the inferior quality, q_C. We find that the expected profit available to the stronger firm under the honest arrangement becomes larger as firms become more asymmetric and thereby reduces the bribe that can be extracted by the corrupt agent. In addition, our analysis shows that while the agent is able to freely manipulate bids in the first-score auction, he is able to do so in the second-score auction only if $\theta_s < \eta_w$, in case the weaker firm is favored. This is in contrast to the symmetric bidder setting in which the agent has unlimited manipulation powers in both the auction formats.

Our results depend crucially on two features of the model – (a) that the agent immediately gets to know the type of the approached firm and (b) that he makes a take-it-or-leave-it offer to the same seller. These attributes help the agent in appropriating the entire surplus from the transaction. If we instead assume that the agent remains uninformed of the approached firm's type, the agent could then either incentivize the firm to truthfully reveal its type, or solve for the optimal bribe offer which maximizes his expected payoff. In either case, the agent will earn a smaller bribe. The take-it-or-leave-it bargaining protocol is used here to keep the probability of detection low, even though it yields an inequitable distribution of the surplus. During the manipulation process, the agent uses his information advantage regarding supplied quality to ensure that the favored firm wins the auction and supplies a minimum quality level (whenever possible). In case the buyer is as adept at verifying quality as the agent, the scope of corruption becomes identical to the one in unidimensional auctions (Arozamena and Weinschelbaum [25], Menezes and Monteiro [26] and Burguet and Perry [27]).

The optimal mechanism recommends that the contract be given to the firm with the highest $\frac{\theta_i^2}{\eta_i}$ in the settings with and without corruption, and therefore, penalizes the stronger firm. The optimal mechanism without corruption requires the selected firm to supply quality $q_i^O = \frac{\theta_i^2}{\eta_i}$. The corresponding mechanism with corruption underscores the limited distortionary powers of the agent and dictates the buyer to (i) correctly deduce the firm who would be approached by the corrupt agent and (ii) to discriminate on the basis of the identity of the winning firm. Under the assumption that the buyer uses a symmetric scoring rule, we show that neither the first-score nor the second-score auction is able to implement the optimal mechanism[23]. Our paper thus highlights the need for additional research regarding the implementation of the optimal mechanism in such settings.

In a related paper Chandel and Sarkar[31] show that the buyer prefers the first-score over the second-score auction when there is no delegation ($x = 0$). This implies that the *Expected Utility Equivalence Result* of Che[4] no longer holds in the presence of asymmetric bidders[24]. Their finding is congruous to that of Maskin and Riley[22], who show that if the strong bidder's type distribution is a "shifted" or a "stretched" version of that of the weaker bidder, the expected revenue from the first-price auction is higher than that from the second[25].

In case the auction is delegated to an agent who is corrupt with probability $x > 0$, we find (i) that the buyer prefers the first-score auction whenever the stronger firm is favored and (ii) the buyer favors the second-score auction whenever the agent enters into a corrupt arrangement with the weaker firm and the probability of corruption is high. For higher levels of bidder asymmetry, the buyer is aware that a corrupt agent prefers to approach the weaker firm. The buyer, therefore, switches from the first-score to the second-score auction when it becomes increasingly likely that the agent is corrupt and that he approaches the weaker firm for a bribe but is unable to manipulate bids in favor

[23]This result is similar to the one found in unidimensional auctions, which states that neither the first-price nor the second-price auction implements the optimal mechanism in an asymmetric firm setting. For implementation of the optimal mechanism in such setups, see Izmalkov[28], Caillaud and Robert[29] and Deb and Pai[30].

[24]Che[4] shows that under a *Naive* Scoring Rule which truly reflects the seller's preferences, the first-score, the second-score and the second-preferred-offer auctions yield the same expected utility to the buyer.

[25]Using a mechanism design approach, Kirkegaard[21] show that the dominance of the first-price auction remains unchanged when the strong bidder's distribution is flatter and more disperse than that of the weaker bidder.

of the preferred firm. We believe that these findings will be of interest to both auction designers and public procurement managers.

In our setup, efficiency dictates that the project be awarded to the supplier with the highest efficiency parameter, θ_i, who may or may not be the one with the highest $\frac{\theta_i^2}{\eta_i}$. In the benchmark model without corruption, we show that the first-score auction is not efficient as it awards the contract to the supplier with lower θ_i with positive probability; the second-score auction, in contrast, is efficient. While the first-score auction continues to be inefficient with delegation, the second-score auction becomes inefficient. Efficiency is restored in the second-score auction if we alter the sequence of events and allow the corrupt agent to approach the winning firm *after* the placement of bids and to offer to manipulate its bid in exchange for a bribe.

As an extension of our model we studied the case where that the procurement agent in addition to being corrupt, is *vengeful*, i.e. if the agent approaches either of the firms for a bribe and the firm refuses, the agent uses his manipulation power to ensure that the firm which turned down his offer does not win the auction afterwards. We find in the first-score auction, the corrupt and vengeful agent always approaches the stronger firm for a bribe, while in the second-score auction, the results are identical to those with a corrupt agent. The preferred scoring rule of the buyer, therefore, remains unchanged.

The manipulation process used in our model ensures that it will be difficult to detect corruption on the basis of payments since the expected payment made in the corrupt arrangement is set equal to that of the honest auction. However, we believe that the difference between the winner's score and the highest losing score offers some hope in this respect. We propose that in addition to announcing the winner's identity, payment to be made and delivered quality, the agent is made to announce the second-highest (highest) score in the first-score (second) auction. Following the empirical literature on the detection of corruption in unidimensional bid auctions (Ingraham[32]), we posit that this difference will be significantly lower in auctions which are run by corrupt agents.

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A Proofs of Propositions

Proof of Proposition 1. For $\lambda\underline{\eta} = \underline{\lambda}$ and $\lambda\eta_i = \lambda_i : \forall : i = s, w$, we have the system of differential equations as:

$$(\phi_i(b) - b)\phi_j'(b) = \phi_j(b) - \underline{\lambda} \tag{A-1}$$
$$s.t. \quad \phi_i(\underline{\lambda}) = \underline{\lambda} \; \forall i = s, w$$

This can be rewritten as

$$(\phi_i(b) - b)(\phi_j'(b) - 1) = \phi_j(b) - \underline{\lambda} - \phi_i(b) + b.$$

Adding the two equations for $i, j = s, w$, we get

$$\frac{d}{db}(\phi_s(b) - b)(\phi_w(b) - b) = 2b - 2\underline{\lambda} \tag{A-2}$$

Integrating this, we obtain

$$(\phi_s(b) - b)(\phi_w(b) - b) = b^2 - 2\underline{\lambda}b + K \tag{A-3}$$

where K is the constant of integration. Substituting $\phi_s(\underline{\lambda}) = \phi_w(\underline{\lambda}) = \underline{\lambda}$, we get $K = \underline{\lambda}^2$.

Therefore equation (A-3) becomes,

$$(\phi_s(b) - b)(\phi_w(b) - b) = (b - \underline{\lambda})^2 \tag{A-4}$$

The highest bid submitted in this auction i.e. $\bar{b}$ is calculated by substituting $\phi_i(\bar{b}) = \lambda_i : \forall i = s, w$.

$$\bar{b} = \frac{\lambda_s\lambda_w - \underline{\lambda}^2}{\lambda_s + \lambda_w - 2\underline{\lambda}} \tag{A-5}$$

or

$$\bar{b} = \lambda\frac{\eta_s\eta_w - \underline{\eta}^2}{\eta_s + \eta_w - 2\underline{\eta}}. \tag{A-6}$$

The inverse bidding strategies for the firms are,

$$\phi_j^{FS}(b) = \underline{\lambda} + \frac{2(b - \underline{\lambda})}{1 - k_j(b - \underline{\lambda})^2} \qquad \forall j = s, w \tag{A-7}$$

or

$$\phi_j^{FS}(b; \lambda) = \lambda\underline{\eta} + \frac{2(b - \lambda\underline{\eta})}{1 - k_j(b - \lambda\underline{\eta})^2} \qquad \forall j = s, w \tag{A-8}$$

where k_j is a constant of integration $\forall j = s, w$. Since $\phi_j(\bar{b}) = \lambda_j$, where $\bar{b}$ is defined in (A-5), we obtain the constants of integration as

$$k_j = \frac{1}{(\lambda_i - \underline{\lambda})^2} - \frac{1}{(\lambda_j - \underline{\lambda})^2} \qquad \forall j = s, w \tag{A-9}$$

or

$$k_j = \frac{1}{\lambda^2}\left(\frac{1}{(\eta_i-\underline{\eta})^2}-\frac{1}{(\eta_j-\underline{\eta})^2}\right) \qquad \forall j=s,w \tag{A-10}$$

Finally, the bidding strategies, obtained by inverting (A-8) are,

$$\beta_j^{FS}(\lambda\theta;\lambda)=\lambda\underline{\eta}+\frac{1}{k_j(\lambda\theta-\lambda\underline{\eta})}\left(-1+\sqrt{1+k_j(\lambda\theta-\lambda\underline{\eta})^2}\right) \qquad \forall j=s,w \tag{A-11}$$

For any firm of type θ_i, the pseudo-valuation is $\lambda\theta_i$ and the bidding score is $\beta_i(\lambda\theta_i)$. The quality bid of this firm is $q_o(\theta_i)=\lambda\theta_i^2$. The price bid can be computed using $\beta_i(\lambda\theta_i)=s(q_o(\theta_i))-p_i$ or $p_i=2\lambda\theta_i-\beta_i(\lambda\theta_i)$. Thus, after substituting for $\beta_i(.)$ in the above, the price and quality offers of each firm in this first-score auction are,

$$q_s^{FS}(\theta;\lambda)=q_w^{FS}(\theta;\lambda)=q_o(\theta;\lambda)=\lambda\theta^2 \tag{A-12}$$

$$p_i^{FS}(\theta;\lambda)=\lambda(2\theta-\underline{\eta})-\frac{1}{k_i\lambda(\theta-\underline{\eta})}\left(-1+\sqrt{1+k_i\lambda^2(\theta-\underline{\eta})^2}\right) \tag{A-13}$$

□

Proof of Proposition 3. Due to the Revelation Principle, we restrict our attention to direct mechanisms. Let $q(\theta), p(\theta), \sigma(\theta)$ denote respectively the vectors denoting quality levels, transfers and probability of being awarded the project. Let, $i=s,w$

$$v_i(\sigma_i(\theta_i,\theta_j),q_i(\theta_i,\theta_j);\theta_i)=\sigma_i(\theta_i,\theta_j)\frac{1}{\theta_i}q_i(\theta_i,\theta_j) \tag{A-14}$$

and let

$$g_i(\theta_i',\theta_i)=E_{\theta_j}[p_i(\theta_i',\theta_j)-v_i(\sigma_i(\theta_i',\theta_j),q_i(\theta_i',\theta_j);\theta_i)]. \tag{A-15}$$

The optimal mechanism then solves

$$\max_{q(\theta),p(\theta),\sigma(\theta)} E_\theta\left[\sigma_s(\theta)V(q_s(\theta))+\sigma_w(\theta)V(q_w(\theta))-p_s(\theta)-p_w(\theta)\right]$$

s.t.

$$g_i(\theta_i,\theta_i)\geq 0 \;\forall\theta_i\in[\underline{\eta},\eta_i]\;\; i=s,w$$

$$g_i(\theta_i,\theta_i)\geq g_i(\theta_i',\theta_i)\;\forall\theta_i,\theta_i'\in[\underline{\eta},\eta_i]\;\; i=s,w$$

$$\sigma_s(\theta),\sigma_w(\theta)\geq 0 \text{ and } \sigma_s(\theta)+\sigma_w(\theta)\leq 1,\;\forall\theta_w\in[\underline{\eta},\eta_w],\forall\theta_s\in[\underline{\eta},\eta_s]. \tag{A-16}$$

Let $U_i(\theta_i) = g_i(\theta_i, \theta_i)$. From equation (A-15) we can write
$E_{\theta_i}[g_i(\theta_i, \theta_i)] = E_{\theta_i} E_{\theta_j}[p_i(\theta_i, \theta_j)] - E_{\theta_i} E_{\theta_j}[v_i(\sigma_i(\theta_i, \theta_j), q_i(\theta_i, \theta_j); \theta_i)]$
$\Rightarrow E_{\theta_i}[U_i(\theta_i)] = E_\theta[p_i(\theta)] - E_\theta[v_i(\sigma_i(\theta), q_i(\theta); \theta_i)]$
$\Rightarrow E_\theta[p_i(\theta)] = E_{\theta_i}[U_i(\theta_i)] + E_\theta[v_i(\sigma_i(\theta), q_i(\theta); \theta_i)]$
Then we can write the objective function as

$$L = E_\theta \left[\sum_{i=s}^{w} \sigma_i(\theta) V(q_i(\theta)) - \sum_{i=s}^{w} v_i(\sigma_i(\theta), q_i(\theta); \theta_i) \right] - \sum_{i=s}^{w} E_{\theta_i}[U_i(\theta_i)]. \quad \text{(A-17)}$$

From the Envelope Theorem,

$$\frac{dU_i}{d\theta_i} = -E_{\theta_j} \left[\frac{\partial}{\partial \theta_i} v_i(\sigma_i(\theta_i, \theta_j), q_i(\theta_i, \theta_j); \theta_i) \right] = E_{\theta_j} \left[\sigma_i(\theta_i, \theta_j) \frac{1}{\theta_i^2} q_i(\theta_i, \theta_j) \right].$$

Integrating, we get

$$U_i(\theta_i) = K + \int_{\underline{\eta}}^{\theta_i} E_{\theta_j} \left[\sigma_i(\widetilde{\theta}_i, \theta_j) \frac{1}{\widetilde{\theta}_i^2} q_i(\widetilde{\theta}_i, \theta_j) \right] d\widetilde{\theta}_i \quad \text{(A-18)}$$

Substituting $\theta_i = \underline{\eta}$, we get $K = U_i(\underline{\eta})$.
$\therefore U_i(\theta_i) = \int_{\underline{\eta}}^{\theta_i} E_{\theta_j} \left[\sigma_i(\widetilde{\theta}_i, \theta_j) \frac{1}{\widetilde{\theta}_i^2} q_i(\widetilde{\theta}_i, \theta_j) \right] d\widetilde{\theta}_i + U_i(\underline{\eta})$. Then,

$$E_{\theta_i}[U_i(\theta_i)] = \int_{\underline{\eta}}^{\eta_i} \left[U_i(\underline{\eta}) + \int_{\underline{\eta}}^{\theta_i} E_{\theta_j} \left[\sigma_i(\widetilde{\theta}_i, \theta_j) \frac{1}{\widetilde{\theta}_i^2} q_i(\widetilde{\theta}_i, \theta_j) \right] d\widetilde{\theta}_i \right] f_i(\theta_i) d\theta_i$$

$$= U_i(\underline{\eta}) + \int_{\underline{\eta}}^{\eta_i} \int_{\widetilde{\theta}_i}^{\eta_i} f_i(\theta_i) E_{\theta_j} \left[\sigma_i(\widetilde{\theta}_i, \theta_j) \frac{1}{\widetilde{\theta}_i^2} q_i(\widetilde{\theta}_i, \theta_j) \right] d\theta_i d\widetilde{\theta}_i$$

$$= U_i(\underline{\eta}) + \int_{\underline{\eta}}^{\eta_i} \left(1 - F_i(\widetilde{\theta}_i)\right) E_{\theta_j} \left[\sigma_i(\widetilde{\theta}_i, \theta_j) \frac{1}{\widetilde{\theta}_i^2} q_i(\widetilde{\theta}_i, \theta_j) \right] d\widetilde{\theta}_i$$

$$= U_i(\underline{\eta}) + \int_{\underline{\eta}}^{\eta_i} \frac{1 - F_i(\widetilde{\theta}_i)}{f_i(\widetilde{\theta}_i)} E_{\theta_j} \left[\sigma_i(\widetilde{\theta}_i, \theta_j) \frac{1}{\widetilde{\theta}_i^2} q_i(\widetilde{\theta}_i, \theta_j) \right] f_i(\widetilde{\theta}_i) d\widetilde{\theta}_i$$

$$= U_i(\underline{\eta}) + \int_{\underline{\eta}}^{\eta_i} \frac{1 - F_i(\widetilde{\theta}_i)}{f_i(\widetilde{\theta}_i)} \left[\int_{\underline{\eta}}^{\eta_j} \sigma_i(\widetilde{\theta}_i, \theta_j) \frac{1}{\widetilde{\theta}_i^2} q_i(\widetilde{\theta}_i, \theta_j) f_j(\theta_j) d\theta_j \right] f_i(\widetilde{\theta}_i) d\widetilde{\theta}_i$$

$$= U_i(\underline{\eta}) + \int_{\underline{\eta}}^{\eta_i} \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \left[\int_{\underline{\eta}}^{\eta_j} \sigma_i(\theta_i, \theta_j) \frac{1}{\theta_i^2} q_i(\theta_i, \theta_j) f_j(\theta_j) d\theta_j \right] f_i(\theta_i) d\theta_i$$

$$= U_i(\underline{\eta}) + \int_{\underline{\eta}}^{\eta_i} \int_{\underline{\eta}}^{\eta_j} \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \left[\sigma_i(\theta_i, \theta_j) \frac{1}{\theta_i^2} q_i(\theta_i, \theta_j) \right] f(\theta) d\theta, \ \because f(\theta) = f_i(\theta_i) f_j(\theta_j)$$

$$= U_i(\underline{\eta}) + E_\theta \left[\sigma_i(\theta_i, \theta_j) \frac{1}{\theta_i^2} q_i(\theta_i, \theta_j) \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right]$$

Substituting $\frac{1-F_i(\theta_i)}{f_i(\theta_i)} = \frac{1}{1/(\eta_i - \underline{\eta})} \left(1 - \frac{\theta_i - \underline{\eta}}{\eta_i - \underline{\eta}}\right) = \eta_i - \theta_i$ into the above expression, we get

$$E_{\theta_i}[U_i(\theta_i)] = U_i(\underline{\eta}) + E_\theta \left[\sigma_i(\theta_i, \theta_j) \frac{1}{\theta_i^2} q_i(\theta_i, \theta_j)(\eta_i - \theta_i) \right] \tag{A-19}$$

Substituting eqns (A-14) and (A-19) into (A-17),

$$\begin{aligned} L &= E_\theta\left[\sum_{i=s}^{w}\sigma_i(\theta)V(q_i(\theta))-\sum_{i=s}^{w}\sigma_i(\theta_i,\theta_j)\frac{1}{\theta_i}q_i(\theta_i,\theta_j)\right. \\ &\qquad \left.-\sum_{i=s}^{w}\sigma_i(\theta_i,\theta_j)\frac{1}{\theta_i^2}q_i(\theta_i,\theta_j)(\eta_i-\theta_i)-\sum_{i=s}^{w}U_i(\underline{\eta})\right] \\ &= E_\theta\left[\sum_{i=s}^{w}\sigma_i(\theta)V(q_i(\theta))-\sum_{i=s}^{w}\sigma_i(\theta_i,\theta_j)q_i(\theta_i,\theta_j)\frac{\eta_i}{\theta_i^2}\right]-\sum_{i=s}^{w}U_i(\underline{\eta}). \end{aligned}$$

Sufficient conditions for incentive compatibility condition (A-16) to hold are that (A-18) holds and that $E_{\theta_j}[\sigma_i(\theta_i,\theta_j)q_i(\theta_i,\theta_j)]$ is non-decreasing. Hence the optimal mechanism solves

$$\max_{q(\theta),p(\theta),\sigma(\theta)} E_\theta\left[\sum_{i=s}^{w}\sigma_i(\theta)V(q_i(\theta))-\sum_{i=s}^{w}\sigma_i(\theta_i,\theta_j)q_i(\theta_i,\theta_j)\frac{\eta_i}{\theta_i^2}\right]-\sum_{i=s}^{w}U_i(\underline{\eta}) \quad \text{(A-20)}$$

s.t. $\sigma_s(\theta),\sigma_w(\theta)\geq 0$ and $\sigma_s(\theta)+\sigma_w(\theta)\leq 1$, $\forall\theta_w\in[\underline{\eta},\eta_w]$, $\forall\theta_s\in[\underline{\eta},\eta_s]$.
In order to maximize (A-20), we ignore the incentive compatibility constraints and set $U_i(\underline{\eta})=0$. The optimal mechanism then requires that $\sigma_i(\theta)=1$ provided $\frac{\theta_i^2}{\eta_i}>\frac{\theta_j^2}{\eta_j}$. To see this, substitute the optimal value of q_i into (A-20) to get the objective function

$$\sigma_i\left[V(q_i)-q_i\frac{\eta_i}{\theta_i^2}\right]=\sigma_i\left[\log q_i-q_i\right]=\sigma_i\left[\log\frac{\theta_i^2}{\eta_i}-\frac{\theta_i^2}{\eta_i}\frac{\eta_i}{\theta_i^2}\right]=\sigma_i\left[\log\frac{\theta_i^2}{\eta_i}-1\right].$$

Unlike the symmetric bidder case, the good is therefore no longer allocated to the supplier with highest θ_i. The optimal auction, in some sense, penalizes bidders with higher maximum valuations. The optimal quality level in that case is $q_i(\theta_i,\theta_j)\in\arg\max_{q_i}\left\{V(q_i(\theta))-q_i(\theta)\frac{\eta_i}{\theta_i^2}\right\}\Leftrightarrow q_i^O=\frac{\theta_i^2}{\eta_i}$. Both the optimal quality $q_i(\theta)$ and the optimal probability $\sigma_i(\theta)$ are non-decreasing in θ_i, such that $E_{\theta_j}[\sigma_i(\theta_i,\theta_j)q_i(\theta_i,\theta_j)]$ is non-decreasing and the incentive constraint is satisfied. □

Proof of Proposition 7. (a) If the agent approaches the stronger firm, he is able to manipulate bids freely in favor of the stronger firm under the contention that the stronger firm has highest $\frac{\theta_i^2}{\eta_i}$. In that case, the optimal auction with corruption should solve

$$\max_{q(\theta),p(\theta),\sigma(\theta)}(1-x)E_\theta\left[\sum_{i=s}^{w}\sigma_i(\theta)V(q_i(\theta))-\sum_{i=s}^{w}p_i(\theta)\right]+x(V(q_C)-p_C) \quad \text{(A-21)}$$

subject to the same constraints as in the proof of Proposition (3). Since the agent ensures that $E_\theta\left[\sum_{i=s}^{w} p_i(\theta)\right] = p_C$, the maximand is equal to

$$E_\theta\left[(1-x)\sum_{i=s}^{w}\sigma_i(\theta)V(q_i(\theta)) - \sum_{i=s}^{w} p_i(\theta)\right] + xV(q_C)$$

$$= E_\theta\left[(1-x)\sum_{i=s}^{w}\sigma_i(\theta)V(q_i(\theta)) - \sum_{i=s}^{w}\sigma_i(\theta_i,\theta_j)q_i(\theta_i,\theta_j)\frac{\eta_i}{\theta_i^2}\right] - \sum_{i=s}^{w} U_i(\underline{\eta}) + xV(q_C)$$

With $V(q_i) = \log q_i$, the solution is the same as the auction with an honest agent, with the difference that the optimal quality is $q_i^O \in \arg\max_{q_i}(1-x)\log q_i - q_i\frac{\eta_i}{\theta_i^2} \Rightarrow q_i^O = (1-x)\frac{\theta_i^2}{\eta_i}$. Substituting into the objective function gives us $\sigma_i\left[(1-x)\log q_i - q_i\frac{\eta_i}{\theta_i^2}\right] = \sigma_i\left[(1-x)\log(1-x)\frac{\theta_i^2}{\eta_i} - (1-x)\frac{\theta_i^2}{\eta_i}\frac{\eta_i}{\theta_i^2}\right] = \sigma_i(1-x)\left[\log(1-x)\frac{\theta_i^2}{\eta_i} - 1\right]$. The contract is therefore allocated to the firm with highest $\frac{\theta_i^2}{\eta_i}$.

(b) On the other hand, assume that the agent approaches the weaker firm for a bribe. In this case, the agent's ability to manipulate bids depends on the allocation rule used by the buyer in the optimal auction. If the buyer uses the same allocation rule as in (a), the agent is able to manipulate bids provided $\frac{\theta_s^2}{\eta_s} < \eta_w \Rightarrow \theta_s < \widehat{\theta} = \sqrt{\eta_s\eta_w}$. In that case, the optimal auction with corruption should solve

$$\max_{q(\theta),p(\theta),\sigma(\theta)}(1-x)E_\theta\left[\sum_{i=s}^{w}\sigma_i(\theta)V(q_i(\theta)) - \sum_{i=s}^{w}p_i(\theta)\right] + x\Pr(\theta_s < \widehat{\theta})[V(q_C)$$

$$-E_\theta(p_w^m(\theta)|\theta_s < \widehat{\theta})] + x\Pr(\theta_s \geq \widehat{\theta})E_\theta\left[V(q_s(\theta))|\theta_s \geq \widehat{\theta}\right] - x\Pr(\theta_s \geq \widehat{\theta})E_\theta\left[p_s(\theta)|\theta_s \geq \widehat{\theta}\right].$$

While manipulating bids, the agent ensures

$$E_\theta\left[\sum_{i=s}^{w}p_i(\theta)\right] = x\Pr(\theta_s < \widehat{\theta})E_\theta(p_w^m(\theta)|\theta_s < \widehat{\theta}) + x\Pr(\theta_s \geq \widehat{\theta})E_\theta(p_s(\theta)|\theta_s \geq \widehat{\theta})$$

where $p_w^m(\theta)$ denotes the (expected) payment made to be weaker firm when bids are manipulated in his favor. In this case, once $\frac{\theta_s^2}{\eta_s} > \eta_w$, the agent cannot manipulate and make the weaker firm the winner. The contract then goes for sure to the stronger firm who is supposed to supply quality $q_s(\theta)$ in exchange for the expected payment $E_\theta[p_s(\theta)|\theta_s \geq \widehat{\theta}]$. The optimal mechanism should therefore solve

$$\max_{q(\theta),p(\theta),\sigma(\theta)} E_\theta\left[\sum_{i=s}^{w}(1-x)\sigma_i(\theta)V(q_i(\theta))-\sum_{i=s}^{w}\sigma_i(\theta_i,\theta_j)q_i(\theta_i,\theta_j)\frac{\eta_i}{\theta_i^2}\right]$$
$$+x\Pr(\theta_s<\widehat{\theta})V(q_C)+x\Pr(\theta_s\geq\widehat{\theta})E_\theta\left[V(q_s(\theta))|\theta_s\geq\widehat{\theta}\right]-\sum_{i=s}^{w}U_i(\underline{\eta}) \qquad \text{(A-22)}$$

s.t. $\sigma_s(\theta),\sigma_w(\theta)\geq 0$ and $\sigma_s(\theta)+\sigma_w(\theta)\leq 1$, $\forall\theta_w\in[\underline{\eta},\eta_w],\forall\theta_s\in[\underline{\eta},\eta_s]$. In this case, the optimal quality $q_w(\theta)$ should solve

$$q_w(\theta_i,\theta_{-i})\in\arg\max_{q_i}\left[\int_{\underline{\eta}}^{\eta_s}\int_{\underline{\eta}}^{\eta_w}\left((1-x)\log q_i(\theta)-q_i\frac{\eta_w}{\theta_w^2}\right)f_wf_s\,d\theta_w\,d\theta_s\right]$$

if the weaker firm is declared as the winner. This would be equivalent to

$$q_w(\theta_i,\theta_{-i})\in\arg\max_{q_i}\log q_i(\theta)-q_i\frac{\eta_w}{\theta_w^2}\Leftrightarrow q_w^O=(1-x)\frac{\theta_w^2}{\eta_w}.$$

If the stronger firm is declared the winner, the optimal quality should solve

$$q_s(\theta_i,\theta_{-i})\in\arg\max_{q_i}\left[E_\theta[(1-x)V(q_i(\theta))-q_i\tfrac{\eta_s}{\theta_s^2}]+x\Pr(\theta_s\geq\widehat{\theta})E_\theta\left(V(q_i(\theta))|\theta_s\geq\widehat{\theta}\right)\right]$$
$$=\int_{\underline{\eta}}^{\eta_s}\int_{\underline{\eta}}^{\eta_w}\left((1-x)\log q_i(\theta)-q_i\tfrac{\eta_s}{\theta_s^2}\right)f_wf_s\,d\theta_w\,d\theta_s+x\int_{\widehat{\theta}}^{\eta_s}\int_{\underline{\eta}}^{\eta_w}\log q_i(\theta)f_wf_s\,d\theta_w\,d\theta_s.$$

which would be equivalent to solving

$$q_s(\theta_i,\theta_{-i})\in\arg\max_{q_i}(1-x)\log q_i(\theta)-q_i\frac{\eta_w}{\theta_w^2}+x\log q_i(\theta)\Leftrightarrow q_s^O=\frac{\theta_s^2}{\eta_s}.$$

In both cases, the contract is allocated to the firm with highest $\frac{\theta_i^2}{\eta_i}$. Finally, the condition for approaching the weaker firm is given by

$$E_{\theta_s}(c(q_C,\theta_s))-E_{\theta_w}[\Pr(\theta_s<\widehat{\theta})c(q_C,\theta_w)]+E_{\theta_s}(g_s(\theta_s,\theta_s))-E_{\theta_w}(g_w(\theta_w,\theta_w))$$
$$\geq\Pr(\theta_s\geq\widehat{\theta})E_\theta(p_s(\theta)|\theta_s\geq\widehat{\theta})$$

such that using the above condition, the buyer can correctly infer which firm the agent will approach for a bribe. □

B Figures

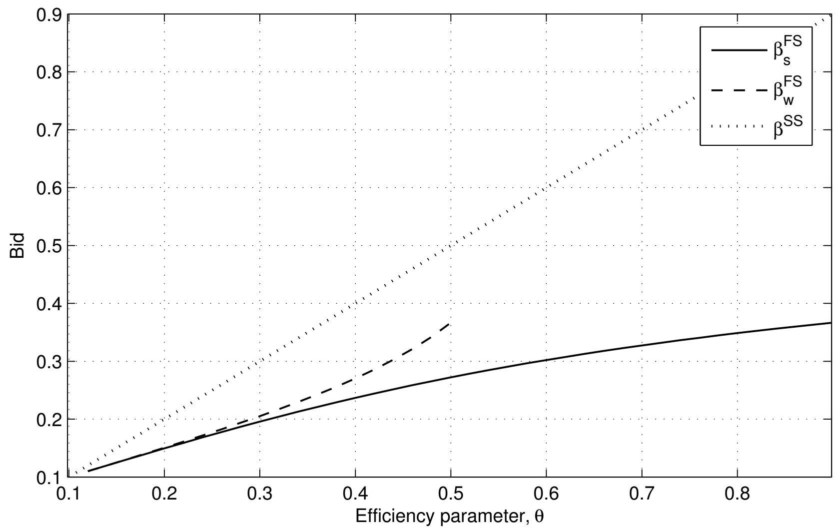
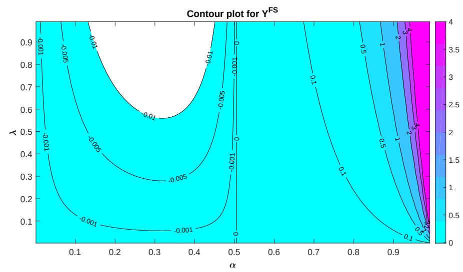
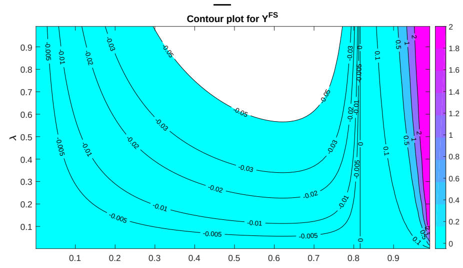


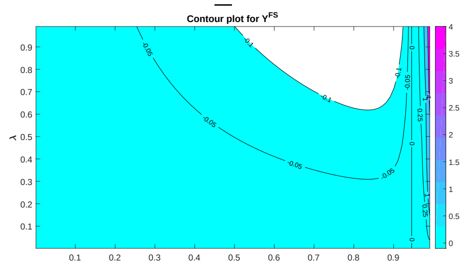
Figure 1: Equilibrium in asymmetric, honest first- and second-score auction.



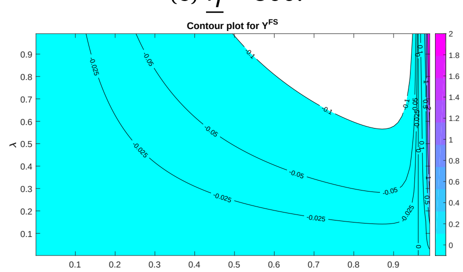
(a) $\underline{\eta} = 10$.



(b) $\underline{\eta} = 50$.

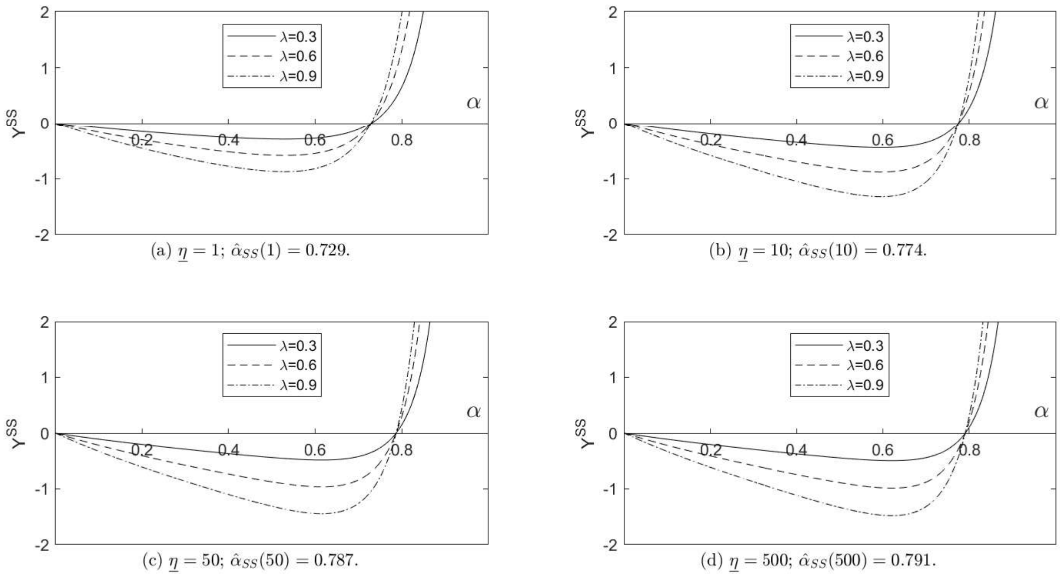


(c) $\underline{\eta} = 500$.



(d) $\underline{\eta} = 1000$

Figure 2: Contour curves of Y^{FS} against α and λ for different values of $\underline{\eta}$



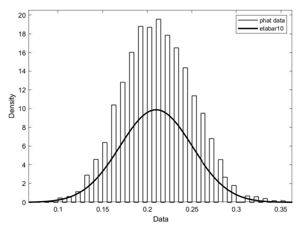
(a) $\underline{\eta} = 1$; $\hat{\alpha}_{SS}(1) = 0.729$.

(b) $\underline{\eta} = 10$; $\hat{\alpha}_{SS}(10) = 0.774$.

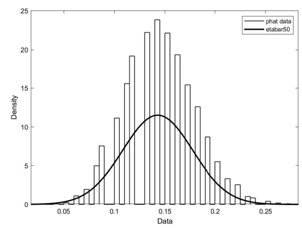
(c) $\underline{\eta} = 50$; $\hat{\alpha}_{SS}(50) = 0.787$.

(d) $\underline{\eta} = 500$; $\hat{\alpha}_{SS}(500) = 0.791$.

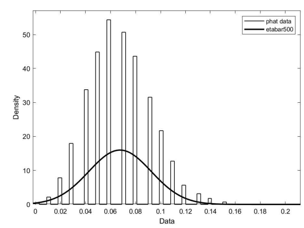
Figure 3: Y^{SS} against α for different $\lambda, \underline{\eta}$



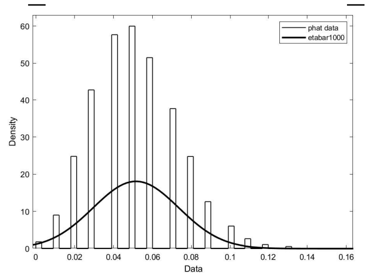
(a) $\underline{\eta} \sim U[0,10]$



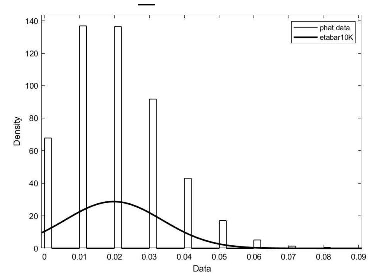
(b) $\underline{\eta} \sim U[0,50]$



(c) $\underline{\eta} \sim U[0,500]$

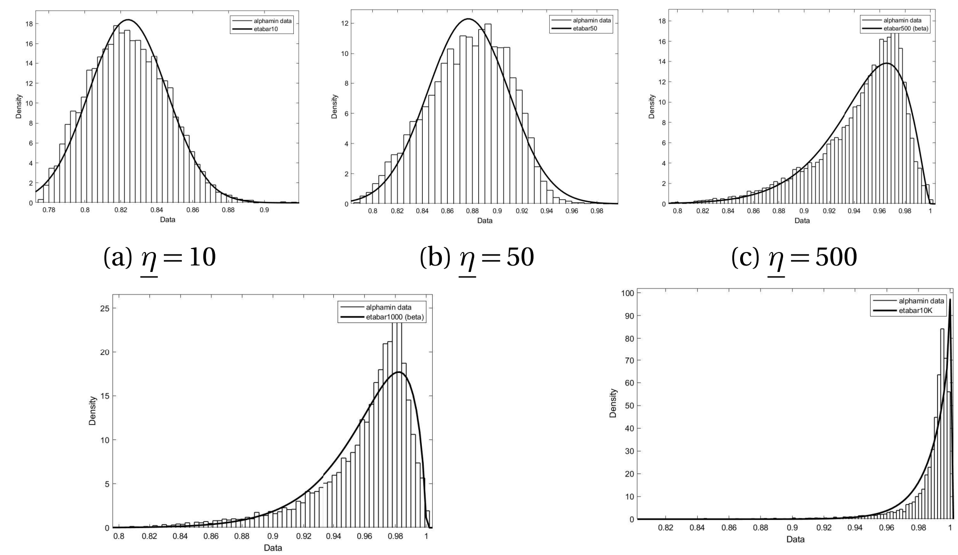


(d) $\underline{\eta} \sim U[0,1000]$



(e) $\underline{\eta} \sim U[0,10000]$

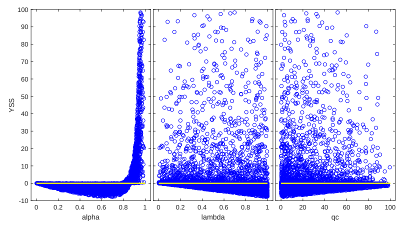
Figure 4: Simulation 1 - Distributions of $\widehat{p}_i$.



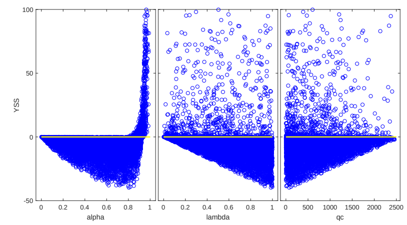
(a) $\underline{\eta} = 10$ (b) $\underline{\eta} = 50$ (c) $\underline{\eta} = 500$

(d) $\underline{\eta} = 1000$ (e) $\underline{\eta} = 10000$

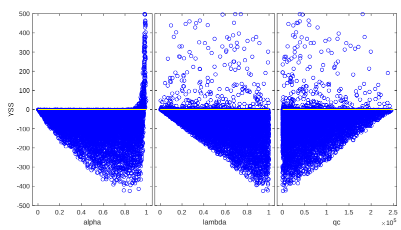
Figure 5: Simulation 2 - Distributions of $\widehat{\alpha}_{SS,i}$. Coefficients of skewness (kurtosis) for the distributions are 0.2269 (2.7141), -0.2293 (2.5078), -1.2020 (4.3393), -1.6828 (6.1601) and -3.9726 (24.5693) respectively.



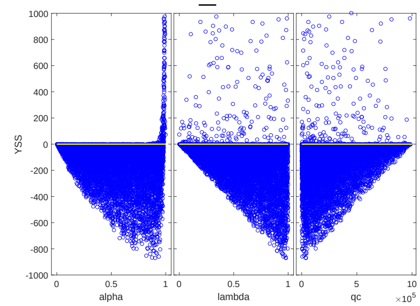
(a) $\underline{\eta} = 10$



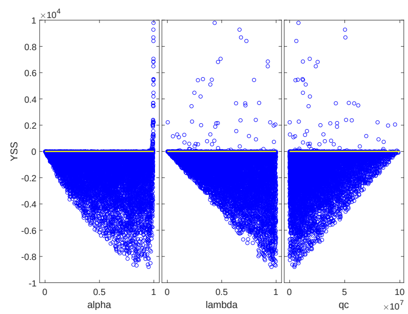
(b) $\underline{\eta} = 50$



(c) $\underline{\eta} = 500$

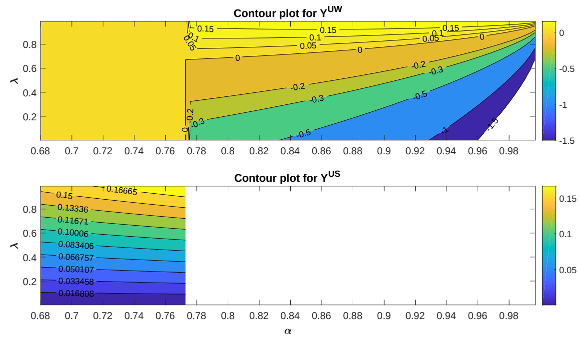


(d) $\underline{\eta} = 1000$

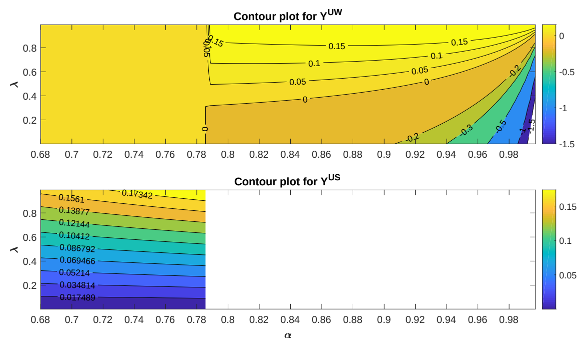


(e) $\underline{\eta} = 10000$

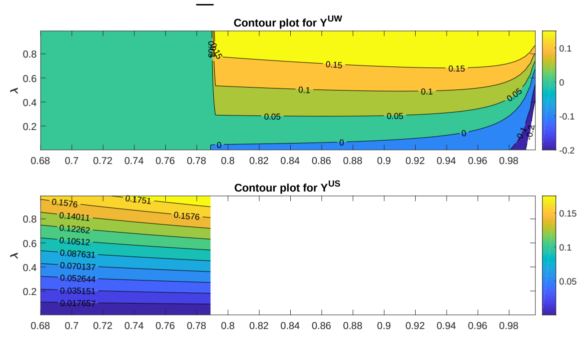
Figure 6: Scatter-plots of Y^{SS} against α,λ and q_C.



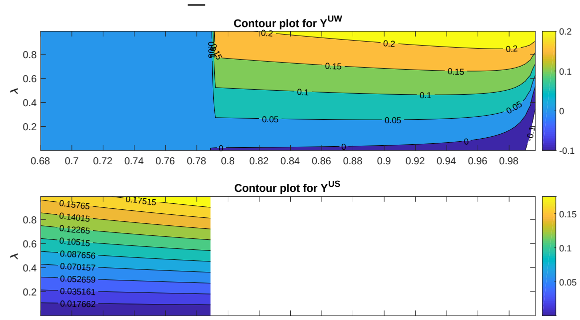
(a) $\underline{\eta} = 10$; $\widehat{\alpha}_{SS} = 0.774$.



(b) $\underline{\eta} = 50$; $\widehat{\alpha}_{SS} = 0.787$.

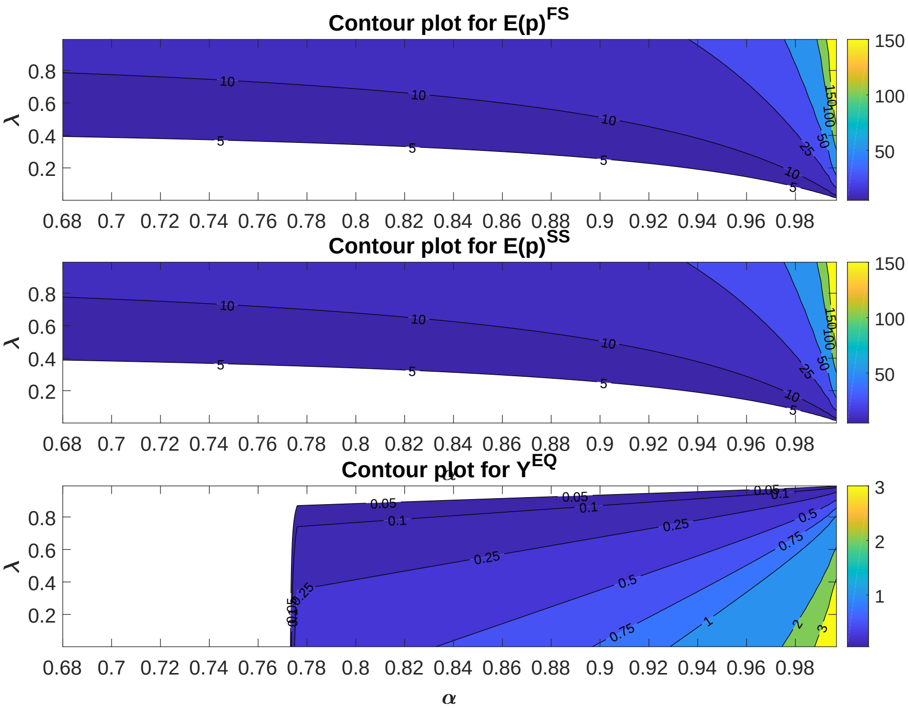


(c) $\underline{\eta} = 500$; $\widehat{\alpha}_{SS} = 0.791$.

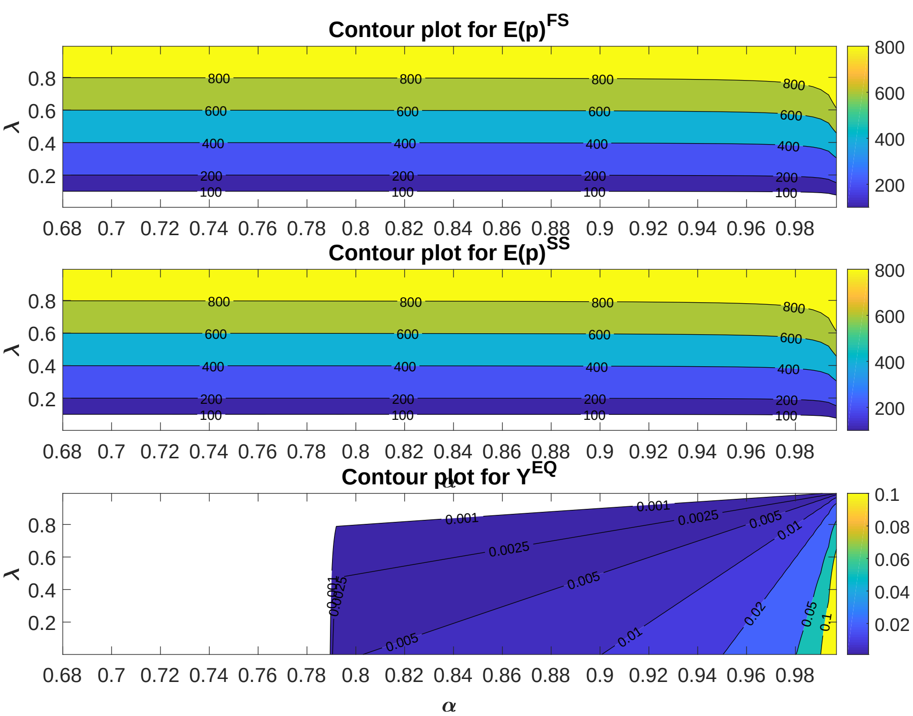


(d) $\underline{\eta} = 1000$

Figure 7: Contour lines of Y^{US}, Y^{UW} against α and λ for different values of $\underline{\eta}$



(a) $\underline{\eta} = 10$



(b) $\underline{\eta} = 1000$

Figure 8: Contour lines of $E(p)^{FS}$, $E(p)^{SS}$ and Y^{EQ} against α and λ for different values of $\underline{\eta}$.

C Tables

Table 1: Simulation Results for Second-Score Auction

Panel A: Simulation results for $\widehat{p}$

			95% CI
$\underline{\eta} \sim U[0,10]$	μ	0.2091	$[0.2083, 0.2099]$
	σ	0.0404	$[0.0398, 0.0409]$
$\underline{\eta} \sim U[0,50]$	μ	0.1429	$[0.1422, 0.1436]$
	σ	0.0346	$[0.0341, 0.0351]$
$\underline{\eta} \sim U[0,500]$	μ	0.0675	$[0.0670, 0.0680]$
	σ	0.0249	$[0.0246, 0.0253]$
$\underline{\eta} \sim U[0,1000]$	μ	0.0515	$[0.0510, 0.0519]$
	σ	0.0220	$[0.0217, 0.0223]$
$\underline{\eta} \sim U[0,10000]$	μ	0.0197	$[0.0194, 0.02]$
	σ	0.0138	$[0.0137, 0.014]$

Panel B: Simulation results for $\widehat{\alpha}_{SS}$

$\underline{\eta}$			95% CI	Minimum $\widehat{\alpha}_{SS}$
1	μ	0.7457	$[0.7456, 0.7460]$	0.7289
	σ	0.0106	$[0.0104, 0.0107]$	$(10,000)$
10	μ	0.8239	$[0.8235, 0.8243]$	0.7750
	σ	0.0217	$[0.0214, 0.0219]$	$(10,000)$
50	μ	0.8767	$[0.8761, 0.8773]$	0.7879
	σ	0.0324	$[0.0320, 0.0329]$	$(10,000)$
500	μ	0.9447	$[0.9440, 0.9454]$	0.7951
	σ	0.0346	$[0.0341, 0.0351]$	(9868)
1000	μ	0.9595	$[0.9591, 0.9603]$	0.7993
	σ	0.0315	$[0.0311, 0.0320]$	(9551)
10000	μ	0.9871	$[0.9867, 0.9875]$	0.8112
	σ	0.0178	$[0.0175, 0.0181]$	(6600)

[1] Minimum $\widehat{\alpha}_{SS}$ obtained by taking $Min_i\ \widehat{\alpha}_{SS,i}$.
[2] In case all 100 observations in a sample comprised $Y^{SS} \leq 0$, the corresponding $\widehat{\alpha}_{SS,i}$ was recorded as "Not a Number (NaN)" and $\widehat{p}_i = 0$.

Table 2: Simulation results for Expected Utility Comparison

Panel A: Simulation results for $\widehat{p}_{f\|w}$			
			95% CI
$\underline{\eta} \sim U[0,10]$	μ	0.0862	$[0.0852, 0.0872]$
	σ	0.0156	$[0.0149, 0.0163]$
$\underline{\eta} \sim U[0,50]$	μ	0.1938	$[0.1919, 0.1957]$
	σ	0.0301	$[0.0288, 0.0315]$
$\underline{\eta} \sim U[0,500]$	μ	0.2820	$[0.2791, 0.2849]$
	σ	0.0466	$[0.0446, 0.0487]$
$\underline{\eta} \sim U[0,1000]$	μ	0.3174	$[0.3140, 0.3208]$
	σ	0.0548	$[0.0525, 0.0573]$
Panel B: Simulation results for $\widehat{p}_w, \alpha > 0.68$			
			95% CI
$\underline{\eta} \sim U[0,10]$	μ	0.3072	$[0.3063, 0.3082]$
	σ	0.0149	$[0.0143, 0.0156]$
$\underline{\eta} \sim U[0,50]$	μ	0.1724	$[0.1718, 0.1732]$
	σ	0.0115	$[0.0110, 0.0120]$
$\underline{\eta} \sim U[0,500]$	μ	0.0993	$[0.0987, 0.0998]$
	σ	0.0093	$[0.0089, 0.0097]$
$\underline{\eta} \sim U[0,1000]$	μ	0.0765	$[0.0760, 0.0771]$
	σ	0.0084	$[0.0080, 0.0088]$