Example 1 of econometric analysis: the Market Model

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The Market Model

- Investors want an equation predicting the return from investing in alternative securities.
- Return is the change in the value of investment over the period it is held.
- The economic problem: formulating the linear model for $E(r)$ of a security.

Defining variables

- $E(r)$ is defined as $(p_1 + d - p_0)/p_0$
  p_0 is the value at the time of making the investment. p_1 is the value at the time of investments. d are any cashflows that have accrued to the investor while holding the investment.
- Investors care about the distribution of $E(r)$.
  One assumption is that $E(r)$ is normally distributed – need to know σ^2.
- σ is called the "risk" of the investment.
  Typically, risk is measured by σ rather than σ^2.
- Investors prefer lower risk. For higher risk, $E(r)$ ought to be higher.
- Lowest risk investment – investment in lending money to the government. This earns the economy's "risk-free return", r_f.
- Risky investments should earn a "risk premium"over r_f:

$$\text{risk premium}_i = E(r_i - r_f)$$

The market model for E(r_i)

- Market model for $E(r_i)$:

$$E(r_i) \quad = \quad r_f + \beta_i E(r_M - r_f)$$

- This model says that there is one factor affecting security returns: the overall market returns, r_M.
  $(r_M - r_f)$ is the "excess return on the market index".
- How the market factor affects returns on a single stock is through β_i.
  If $\beta_i = 0$, the risk premium for investing in i, $E(r_i - r_f) = 0$!
- Once we know β_i for a stock, and the "expected excess returns on the market index", we can calculate $E(r_i)$.

Understanding β_i

- This model is called the "Single Index Market Model" or the "market model". The econometric model is written as:

$$r_i = r_f + \beta_i(r_M - r_f) + \epsilon_i$$

- From this model, we see that risk of the security, measured by it's variance, can be separated into two parts :

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_\epsilon^2$$

  (By definition, the variance of the risk-free rate is zero).
- The term $\beta_i^2 \sigma_M^2$ term captures *systematic risk* in the risk of stock *i*.
- σ_ϵ^2 is the unsystematic risk.
  When this security is held in a portfolio with other securities, the σ_ϵ^2 part is driven down close to zero.

Estimating β_i

- In the model:

$$r_i = r_f + \beta_i(r_M - r_f) + \epsilon_i$$

the OLS estimator for β_i is

$$\hat{\beta}_i = \mathrm{cov}_{i,M}/\sigma_M^2$$

Data and analysis for Single Index Market Model estimation of Beta

Data to estimate beta$_i$

- Time period for analysis: Jan 1 2003 to Dec 31 2007
- Frequency of data: monthly
- Fully diversified portfolio: NSE-50 market index.
- Any stock – it is not important that it is in the stock index or not.
  Our examples: Infosys Technologies, ICICI Bank
- Calculate monthly returns for the stocks and the index:

  $$r_i = 100 * (\log P_1 - \log P_0)$$

  where P_0 is the price at the start of the month. P_1 is the price at the end of the month.
- r_f is the three-month interest rate set by the RBI.
- Use OLS to estimate beta for Infosys and ICICI Bank assuming the single index market model is the best model for $E(r_i)$.

Analysis framework

- We want to estimate the following linear relationship:

$$r_i = r_f + \beta_i(r_M - r_f)$$

  Equivalent statement:

$$r_i - r_f = \beta_i(r_M - r_f)$$

- Econometric model form 1:

$$Y_i = \alpha + \beta_i X_i + \epsilon_i$$

- What are the null hypotheses?

$$\begin{aligned} 1. Y_i &= r_i, X_i = r_M - r_f \\ H_0 : \beta_i &= 1; H_a : \beta \neq 1 \\ H_0 : \alpha &= r_f; H_a : \alpha \neq r_f - \text{ not well posed} \\ 2. Y_i &= r_i - r_f, X_i = r_M - r_f \\ H_0 : \beta_i &= 1; H_a : \beta \neq 1 \\ H_0 : \alpha &= 0; H_a : \alpha \neq 0 \end{aligned}$$

- Can we use OLS to estimate this model?

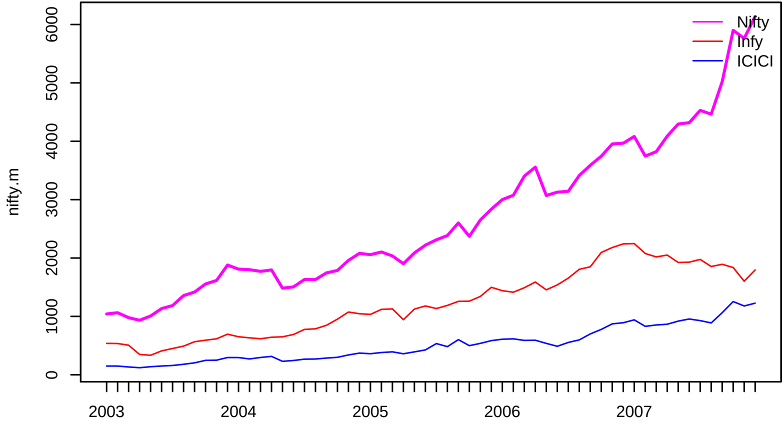
Testing the data for OLS assumptions

- Correct unconditional distribution of Y – specifically for OLS, normality of Y.
- iid-ness of ϵ – serial dependence and homoskedasticity.
- Correct conditional distribution of Y – do we have the right model of Y on X?
- Generally the tests do not use the LR form because the test specified alternative.

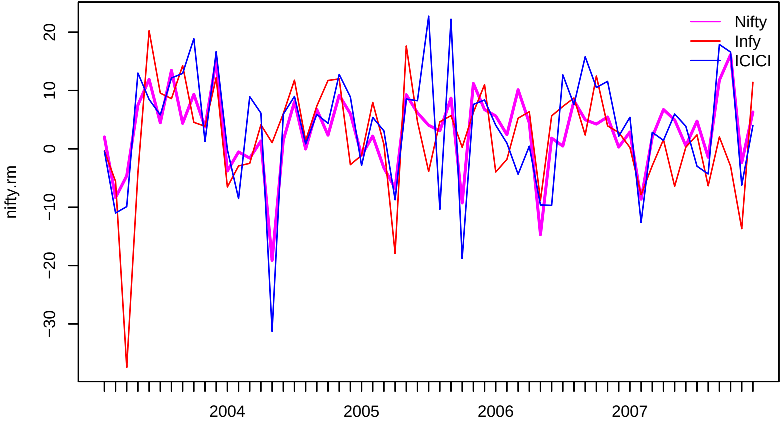
Testing for unconditional distribution of Y

- The distribution of Y is generally tested for normality.
- First question: Y, or a monotonically transformed version of Y?
- Use graphical tools, statistical tools

Distribution of prices



Distribution of ∂ returns



Statistical tests

- Check for the Skewness of the data – should be 0

$$N\frac{\hat{\text{skew}}^2}{6} \sim \chi^2(1) = \chi^2_{\text{skew}}$$

- Check for the Kurtosis of the data – should be 3

$$N\frac{(\hat{\text{kurtosis}} - 3)^2}{24} \sim \chi^2(1) = \chi^2_{\text{kurtosis}}$$

- **Jarque-Bera** test: jointly test for the skewness and the kurtosis as:

$$\chi^2_{\text{skew}} + \chi^2_{\text{kurtosis}} = \chi^2(2)$$

Summary statistics for the series, tests for normality

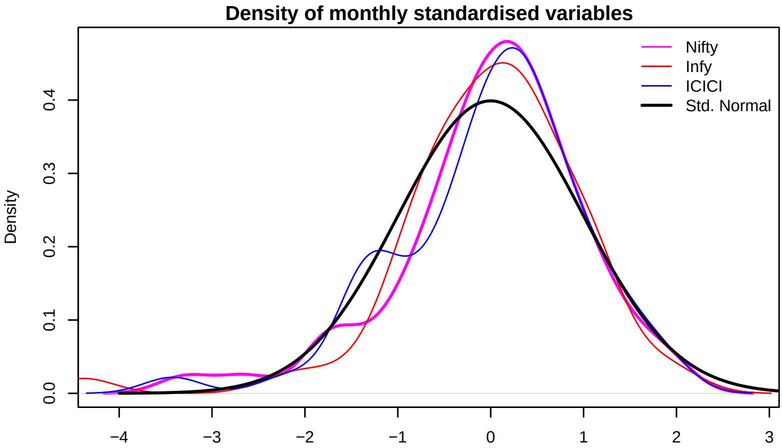
- Summary statistics for monthly data

	Nifty	Infy	ICICI Bank
Mean	3.0061	2.0449	3.5629
Std. Dev.	6.7305	9.0119	10.1755
Skewness	-0.8148	-1.3728	-0.7185
	(6.70)	(18.85)	(5.16)
Kurtosis	4.1742	7.6354	3.9752
	(3.45)	(53.72)	(2.38)

Number of observations = 60 observations

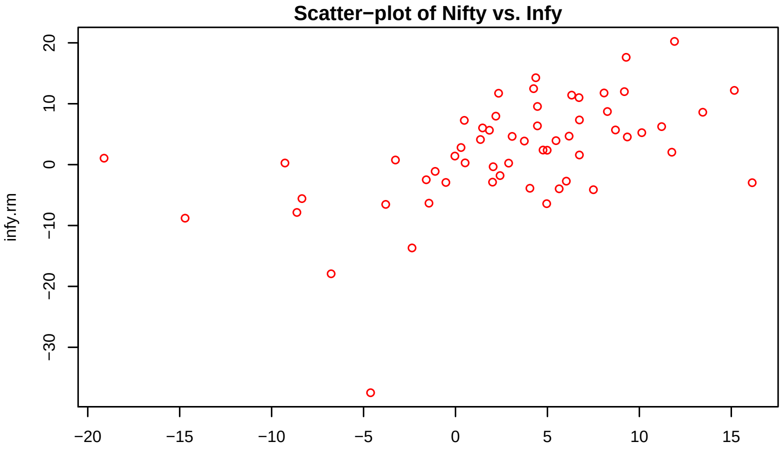
- $\chi^2(1),$ 0.05% level of significance = 3.84, 0.01% = 6.63
- $\chi^2(2),$ 0.05% level of significance = 5.99, 0.01% = 9.21
- Significantly different from a normal distribution – will it affect the OLS estimation? How?
- Will MLE be a better estimation tool here? Not if we don't know the exact form of the distribution.

Density plot for the three series, standardised

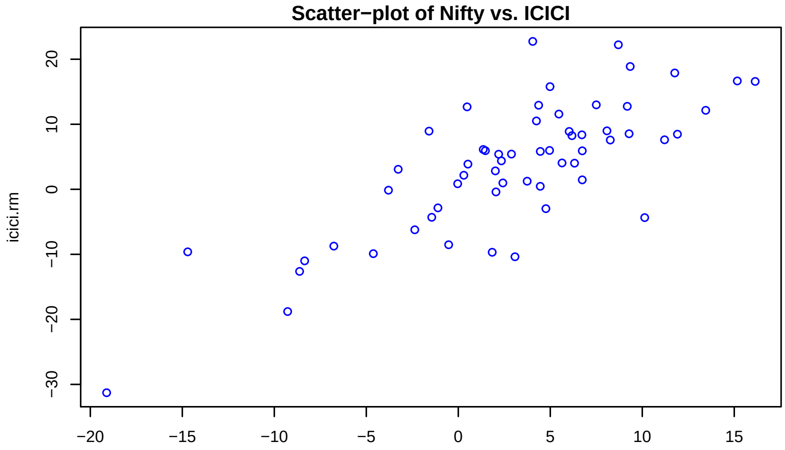


Is there any relationship between the stock and market returns?

Distribution of Nifty returns vs. Infy returns



Distribution of Nifty returns vs. ICICI Bank returns



Relationship between Stock and Market returns

- Looks like the relationship is positive.
  As expected.
- Variance-covariance matrix for the three returns:

Covariance matrix			Correlation matrix		
45.30	31.70	53.28	1.000	0.523	0.778
	81.21	38.04		1.000	0.415
		103.54			1.000

- In the case of Infy vs. Nifty, there appears to be one outlier (April 2003, Infy returns -37.45%, Nifty -4.62%)

Model estimation for Infy beta

- $r_{\text{infy}} - r_f = \alpha_0 + \beta_{\text{infy}}(r_{\text{Nifty}} - r_f) + \epsilon$
- Regression Results:

	Estimate	Std. Error	t-value	Prob value
(Intercept)	-1.6413	1.0974	-1.496	0.140
Market returns	0.7474	0.1516	4.930	7.45e-06
	F-stat(1, 57) = 24.31 prob value = 7.5e-6 R-squared = 0.2989 Adjusted R-squared: 0.2866 Residual Std. Err.: 7.7270			

- Model: $E(r_{\text{infy}} - r_f) = 0.747E(r_{\text{Nifty}} - r_f)$

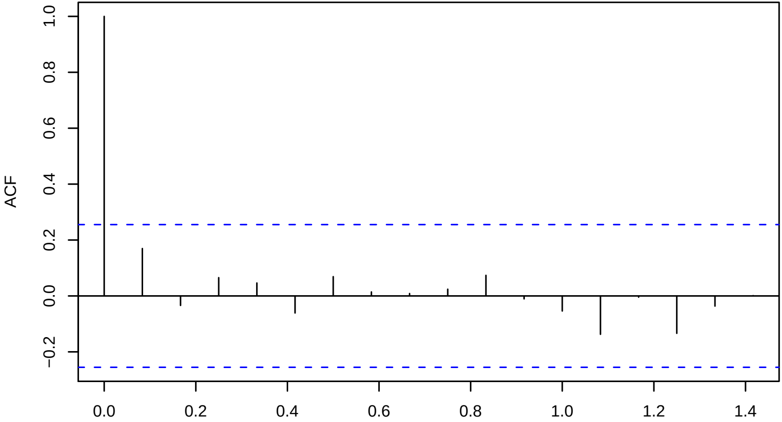
Error analysis

- Characteristics of ϵ.

	Y	ϵ
Mean	2.0449	0
Std. Dev.	9.0119	7.7572
Skewness	-1.3728	-1.3156
	(18.85)	(17.31)
Kurtosis	7.6354	7.3402
	(53.72)	(47.09)
Number of observations = 60 observations		

- Correlation between $(r_M, \epsilon) = -1.6e - 17$
- Serial dependence in ϵ: autocorrelation coefficient function of ϵ.

Serial dependence in errors



HW

- Find out the beta for ICICI Bank for the same period, same frequency of data.
  Compare the regression results to those for Infosys Technologies. What can you say about the risk of ICICI Bank wrt Infosys Tech.?
- Read up the literature on the linkages between the financial sector and the real economy.
  The seminar paper that started this work was **Stock Returns, Real Activity, Inflation, and Money**, by *Eugene F. Fama*, The American Economic Review, Vol. 71, No. 4 (Sep., 1981), pp. 545-565