Multinomial Logit Models

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INTRODUCTION

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- 2 types of unordered models-
  1. Sequential logit
  2. Multinomial logit
- 3 types of Multinomial Logit models-
  1. Generalized logit
  2. Conditional logit
  3. Mixed logit

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- Errors are independently and identically distributed.

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- Assuming $U_i \sim N\ (0\ ,1)$, i.e., probit, let Π_{jk} denote the probability that individual j chooses alternative k, let X_j represent the characteristics of individual j, and let Z_{jk} be the characteristics of the kth alternative for individual j.

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- In fitting such a model, one has to estimate $m - 1$ sets of regression coefficients by setting β_m=0.

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- The impact of a variable on the choice probabilities derives from the difference of its values across the alternatives.

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- It is needed for investigating consumer choice in more detail.

MIXED LOGIT MODEL AS GENERALIZED LOGIT MODEL

- Now as assumed individuals have m choices, the probability of the jth choice is:

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- Here X_i includes two types of information:
  1. The individual socio- economic characteristics, eg. age, income, sex etc.
  2. The choice characteristics. Suppose the m choices retain no different occupations. Then X_i includes the characteristics of all the m occupations. If for the jth choice some of the occupation characteristics are irrelevant then, we simply set the corresponding co-efficient of j to zero.

- Occupation choice $\forall m$ occupations   Socio economic change

$$X_i = \left[\begin{array}{cccccccc} X_{01i} & X_{02i} & X_{03i} & X_{04i} & X_{05i} & \vdots & X_{s1i} & X_{s2i} & X_{s3i} \end{array} \right]$$

Suppose for an occupation 1 only characteristics 1,2,3 are relevant, for occupation 2, only 2,3,5 are relevant. Then

$$\beta_1 = \left[\begin{array}{ccccccccc} \beta_{01} & \beta_{02} & \beta_{03} & 0 & 0 & \vdots & \beta_{16} & \beta_{17} & \beta_{18} \end{array} \right]$$

$$\beta_2 = \left[\begin{array}{ccccccccc} 0 & \beta_{02} & \beta_{03} & 0 & \beta_{05} & \vdots & \beta_{26} & \beta_{27} & \beta_{28} \end{array} \right]$$

Given this specification:

$$\frac{P(Y_i = j|X_i)}{P(Y_i = i|X_i)} = \frac{e^{\beta_j' X_i}}{e^{\beta_i' X_i}} = e^{(\beta_j' - \beta_i') X_i}$$

Continued...

Therefore the relative probability between j and i depends:

1. Only on the difference of β_j and β_i, hence Normalization.

   $$\beta_1 = 0 \Rightarrow e^{\beta_i' X_i} = e^0 = 1 \Rightarrow P(Y_i = j|X_i) = \frac{e^{\beta_j' X_i}}{1 + \sum_{j=2}^{m} e^{\beta_j' X_i}}.$$

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2. Independence of irrelevant alternatives. Suppose m_1 and m_2- 2 choices. The individual is indifferent between the choices, $\frac{P(1)}{P(2)} = 1 \Rightarrow P(1) = P(2) = 0.5$. But say, The individual faces choices m_{11} and m_{12} in m_1. The multinominal logit model reads as, m_{11}, m_{12} and m_2. $\Rightarrow \frac{P(1)}{P(2)} = 1$, $\frac{P(1)}{P(3)} = 1$ and $\frac{P(2)}{P(3)} = 1 \Rightarrow P(1) = P(2) = P(3) = \frac{1}{3}$. In reality $P(1) = 0.25$, $P(2) = .25$, $P(3) = 0.5$. The odds ratio between alternative 1 and 3 is 1:1 in multinomial logit structure, but it is actually 1:2 $\Rightarrow$ inconsistency & depends on the fact that 1 and 2 are correlated choices $\Rightarrow$ use Nested Logit Models.

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- Now we use Newton-Raphson iterative method to estimate the parameters,

$$\hat{\beta}_j = \hat{\beta}_{j-1} - [E(\frac{\partial^2 logL}{\partial\beta\partial\beta'})]^{-1}_{\hat{\beta}_{j-1}} \frac{\partial logL}{\partial\beta}|_{\hat{\beta}_{j-1}}$$

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- The likelihood function is globally concave and therefore guarantees the global maximum. The variance-covariance matrix is given by $[E[\frac{-\partial^2 logL}{\partial\beta\partial\beta'}]]^{-1}$, which can be used to do testing and inference.

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Table : Candy Choice GL Model: Subject Preferences

		Candy		
Gender	Age	Chocolate	Lollipop	Sugar
Boy	Child	2	13	3
Boy	Teenager	10	9	3
Girl	Child	3	9	1
Girl	Teenager	8	0	1

- The logits being modeled are:

$$log([\frac{Pr(Candy = lollipop)}{Pr(Candy = chocolate)}]) = b_{10} + b_{11}(gender = boy) + b_{12}(age = teenager)$$

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- Reference category: Chocolate.
- Reference levels (set=0): Girl for Gender and Child for Age.

Analysis of Maximum Likelihood Estimates

Parameter		Candy	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept		lollipop	1	0.7698	0.5782	1.7722	0.1831
Intercept		sugar	1	-0.9033	0.8664	1.0869	0.2972
Gender	boy	lollipop	1	1.5758	0.7569	4.3347	0.0373
Gender	boy	sugar	1	1.5261	1.0158	2.2570	0.1330
Age	teenager	lollipop	1	-2.6472	0.7572	12.2212	0.0005
Age	teenager	sugar	1	-1.7416	0.9623	3.2754	0.0703

Odds Ratio Estimates

Effect	Candy	Point Estimate	95% Wald Confidence Limits	
Gender boy vs girl	lollipop	4.835	1.097	21.313
Gender boy vs girl	sugar	4.600	0.628	33.686
Age teenager vs child	lollipop	0.071	0.016	0.313
Age teenager vs child	sugar	0.175	0.027	1.155

Testing Global Null Hypothesis: $\beta = 0$

Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	18.9061	4	0.0008
Score	16.9631	4	0.0020
Wald	12.8115	4	0.0122

Type III Analysis of Effects

Effect	DF	Wald Chi-Square	Pr > ChiSq
Gender	2	4.7168	0.0946
Age	2	12.2325	0.0022

Linear Hypothesis Testing Results

Label	Wald Chi-Square	DF	Pr > ChiSq
Test_1	0.0027	1	0.9582
Test_2	1.1585	1	0.2818

Gender Least Squares Means(Prediction)

Candy	Gender	Estimate	Standard Error	z Value	Pr > |z|	Mean
chocolate	boy	0.2480	0.5702	0.43	0.6636	0.2193
chocolate	girl	1.7741	0.8127	2.18	0.0290	0.5733
lollipop	boy	1.2701	0.4687	2.71	0.0067	0.6095
lollipop	girl	1.2203	0.8206	1.49	0.1370	0.3295

Age Least Squares Means(Prediction)

Candy	Age	Estimate	Standard Error	z Value	Pr > |z|	Mean
chocolate	child	0.1403	0.7067	0.20	0.8427	0.1511
chocolate	teenager	1.8819	0.6583	2.86	0.0043	0.6717
lollipop	child	1.6980	0.5579	3.04	0.0023	0.7175
lollipop	teenager	0.7924	0.6965	1.14	0.2552	0.2260

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- GL model examines the relationship between choice of transportation and age.

Snapshot of dataset "choice"

AutoTime	PlaneTime	Trantime	Age	Chosen
10	4.5	10.5	32	Plane
5.5	4	7.5	13	Auto
4.5	6	5.5	41	Transit
3.5	2	5	41	Transit
1.5	4.5	4	47	Auto
10.5	3	10.5	24	Plane
7	3	9	27	Auto
9	3.5	9	21	Plane
4	5	5.5	23	Auto
22	4.5	22.5	30	Plane
7.5	3.5	10	58	Plane
11.5	3.5	11.5	36	Transit
3.5	4.5	4.5	43	Auto
12	3	11	33	Plane
18	5.5	20	30	Plane
23	5.5	21.5	28	Plane
4	3	4.5	44	Plane

Maximum Likelihood Analysis of Variance

Source	DF	Chi-Square	Pr > ChiSq
Intercept	2	1.72	0.4238
Age	2	1.20	0.5478
Likelihood Ratio	34	42.18	0.1583

Analysis of Maximum Likelihood Estimates

Parameter	Function Number	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	3.0449	2.4268	1.57	0.2096
	2	2.7212	2.2929	1.41	0.2353
Age	1	-0.0710	0.0652	1.19	0.2762
	2	-0.0500	0.0596	0.70	0.4013

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Table : Travel Choice- GL predictions

Age	$exp(\beta' X_j)$	*Prob*(*Auto*)	$exp(\beta' X_j)$	*Prob*(*Plane*)
20	5.0779	0.5156	5.5912	0.41408544
30	2.496	0.253	3.3912	0.251155515
40	1.2274	0.1246	2.0569	0.15233352
50	0.6034	0.0612	1.2475	0.0923
60	0.2966	0.0301	0.7566	0.056
70	0.1458	0.0148	0.4589	0.0339
Total=	9.847	1	13.5026	1

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- SUBJECT Variable is created to specify the basis of censoring- refers to the individuals numbering 1,2,...,10.

- The estimation result for probabilities is:
$$p_j = \frac{exp(1.386294DARK_j - 2..197225SOFT_j + 0.8472298NUTS_j)}{\sum_{j=1}^{8} exp(1.386294DARK_j - 2..197225SOFT_j + 0.8472298NUTS_j)}$$

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- **Analysis of Maximum Likelihood Estimates**

Parameter	DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio
dark	1	1.38629	0.79057	3.0749	0.0795	4.000
soft	1	-2.19722	1.05409	4.3450	0.0371	0.111
nuts	1	0.84730	0.69007	1.5076	0.219	2.333

Table : Candy Choice- CL predictions

j	Dark	Soft	Nuts	$exp(\beta' X_j)$	p_j
1	0	0	0	1	0.054003
2	0	0	1	2.333334	0.126006
3	0	1	0	0.111056	0.005997
4	0	1	1	0.25913	0.013994
5	1	0	0	3.999999	0.216011
6	1	0	1	9.3333310	0.504025
7	1	1	0	0.444223	0.023989
8	1	1	1	1.036521	0.055975
			Total	=18.51759	1

Linear Hypothesis Testing Results

Label	Wald Chi-Square	DF	Pr > ChiSq
test1	7.4199	2	0.0245
test2	5.8526	2	0.0536

- To apply CL, one need to rearrange the data to assign failure time for each alternative.

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Subject	Mode	TravTime	Choice	Auto	Plane	AgeAuto	AgePlane
1	Auto	10	2	1	0	32	0
1	Plane	4.5	1	0	1	0	32
1	Transit	10.5	2	0	0	0	0
2	Auto	5.5	1	1	0	13	0
2	Plane	4	2	0	1	0	13
2	Transit	7.5	2	0	0	0	0
3	Auto	4.5	2	1	0	41	0
3	Plane	6	2	0	1	0	41
3	Transit	5.5	1	0	0	0	0
4	Auto	3.5	2	1	0	41	0
4	Plane	2	2	0	1	0	41
4	Transit	5	1	0	0	0	0

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Parameter	DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio	Label
TravTime	1	-0.26549	0.10215	6.7551	0.0093	0.767	TravTime

Table : Travel Choice- CL predictions

j		$exp(\beta' X_j)$	p_j
Auto	4.5	0.302793	0.382335
Plane	10.5	0.061566	0.077739
Transit	3.2	0.4276	0.539926
	Total	=0.791959	1

- Incorporates both types of variables: age and travel time.

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- New variables AgeAuto and AgePlane, each of which represent the products of the individual's age and his failure time for each choice.

Analysis of Maximum Likelihood Estimates

Parameter	DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio	Label
Auto	1	2.50069	2.39585	1.0894	0.2966	12.191	Auto
Plane	1	-2.77912	3.52929	0.6201	0.4310	0.062	Plane
AgeAuto	1	-0.07826	0.06332	1.5274	0.2165	0.925	AgeAuto
AgePlane	1	0.01695	0.07439	0.0519	0.8198	1.017	AgePlane
TravTime	1	-0.60845	0.27126	5.0315	0.0249	0.544	TravTime

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- Such models has widespread applications in the study of consumer preferences, levels of academic achievements, gender based differences in outcomes, medical research and various areas of behavioural economics.
- The assumptions underlying the Multinomial Logit Model often do not hold in practice- Independence of irrelevant alternatives (wayout Nested Logit) and Influences of past choices (wayout Multinomial probit).

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Thank you...