Indira Gandhi Institute Of Research Development

ECONOMETRICS II

SEEMINGLY UNRELATED REGRESSION

INSTRUCTOR :

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Structure of the Presentation

- Introduction and Motivation
- Illustration via a Model
- Estimation of the model
- Results and Interpretation
- First Order Autoregressive Process and Estimation
- SUR with Singular variance matrices

INTRODUCTION

- The SUR system was first proposed by Arnold Zellner in 1962
- It represents individual relationships →several regression equations and possibility of different number of explanatory variables for each equation (Seemingly Unrelated)
- The system of equations →related through the correlation in error terms
- Source of correlation in error terms→ shocks, government policy etc

MOTIVATION

- Two main motivations for SUR, Roger Moon,et al (2006):
- Efficiency gains with combined information set from the system of equations
- Impose/test restrictions on parameters of different equations
- The system has various advantages:
- Simple specification of model
- Saves degree of freedoms
- Wide applications: Effect of government policy, global macroeconomic downturn/upswing etc

BASIC SUR MODEL: ILLUSTRATION

- 3 firms from automotive sector: Ashok Leyland, Mahindra and Mahindra and Tata Motors Limited
- Investment = f (Market Capitalization, net fixed assets)
- $I_{it} = \mathcal{B}_{i1} X_{it1} + \mathcal{B}_{i2} X_{it2} + \epsilon_{it}$ i= 1,2,3 and t=1995-2012
- X_{it} = Market Capitalization
- X_{it2} = Net fixed assets
- Same segment → government policy, sector specific shocks (input prices example: steel) should affect all 3 companies over the same period (Contemporaneous Correlation)
- Data sources: prowess 4.12, Moneycontrol.com, National Stock Exchange (for data on number of outstanding shares and closing prices)

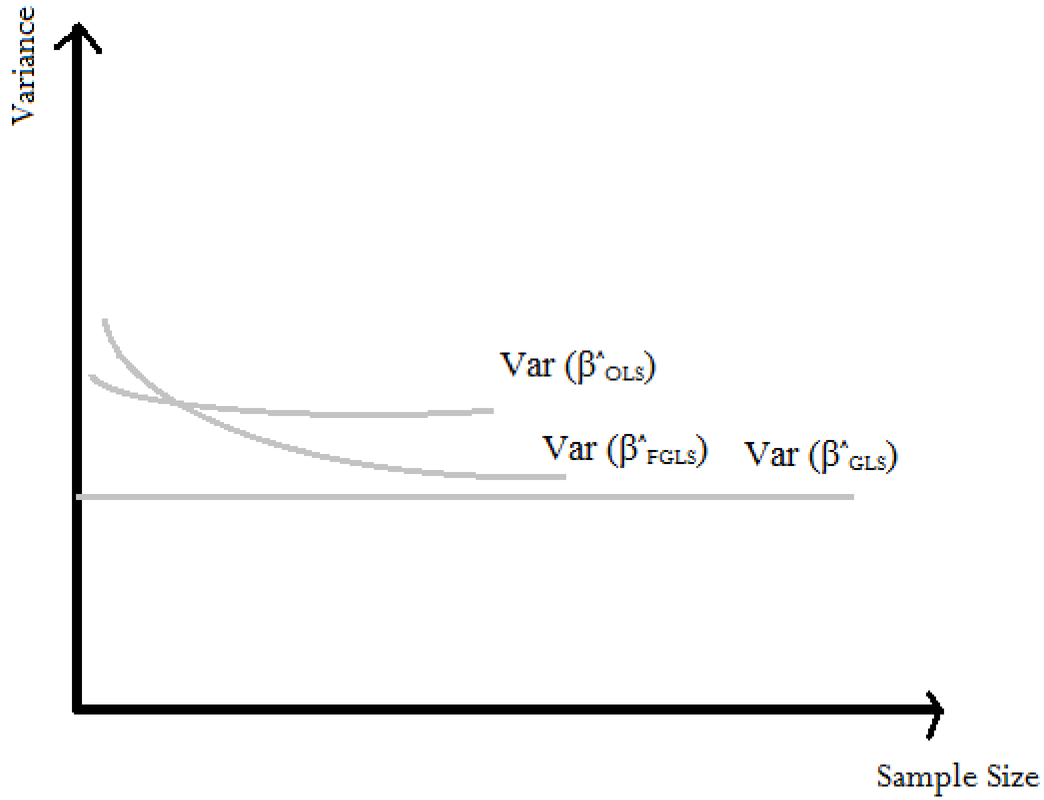
Estimation Assumptions

- Explanatory Variables
- Non stochastic and fixed
- Full Rank (Invertibility)
- Error Terms
- $E(\epsilon_{it}) = 0$ for all i,t
- $E(\epsilon_{it}\epsilon_{is}) = \sigma_{ii}$ if t=s, i=1,2,3
  $= 0$ if $t \neq s$
- $E(\epsilon_{it}\epsilon_{js}) = \sigma_{ij}$ if t=s for $i \neq j$
  $= 0$ if $t \neq s$ for $i \neq j$
- Explanatory Variables & Error Terms
- No correlation between the two → Ensures Consistent $\hat{\beta}_{OLS}$
- Note: Possibility of first order autoregressive Process

Estimation: Points to Remember

Cases	Homoscedasticity within individual	Hetroscedasticity across individuals	Contemporaneous correlation	BLUE
(a)	✔	✘	✘	$\hat{\beta}_{OLS}$
(b)	✔	✔	✘	$\hat{\beta}_{GLS}$ (boils to $\hat{\beta}_{OLS}$)
(c)	✔	✔	✔	$\hat{\beta}_{GLS}$

Relative Efficiency



Results: Ashok Leyland commands.docx

Variables	$\hat{\beta}_{OLS}$ (a)	$\hat{\beta}_{GLS} \leftrightarrow \hat{\beta}_{OLS}$ (b)	Standard Error (b)	$\hat{\beta}_{GLS}$ (c)	Standard Error (c)
Intercept	-1650.7	-1650.7	617.2134	-1993.97	599.426
MktCap	0.08389	0.08389	0.024884	0.080975*	0.019980
NFA	0.18398	0.18398	0.045824	0.212450 *	0.039274

Results: Mahindra and Mahindra

Variables	$\hat{\beta}_{OLS}$ (a)	$\hat{\beta}_{GLS} \leftrightarrow \hat{\beta}_{OLS}$ (b)	Standard Error (b)	$\hat{\beta}_{GLS}$ (c)	Standard Error (c)
Intercept	-4540.26	-4540.26	2466.456	-2828.49	2395.17
MktCap	0.122712	0.122712	0.026082	0.144008*	0.022897
NFA	1.341665	1.341665	0.243745	1.085418*	0.215314

Results: Tata Motors

Variables	$\hat{\beta}_{OLS}$ (a)	$\hat{\beta}_{GLS} \leftrightarrow \hat{\beta}_{OLS}$ (b)	Standard Error (b)	$\hat{\beta}_{GLS}$ (c)	Standard Error (c)
Intercept	-38640.5	-38640.5	13368.10	-32791.8	12980.3
MktCap	-0.11165	-0.11165	0.075409	-0.03413	0.058701
NFA	2.66927	2.669274	0.391279	2.241070*	0.329630

Commands

- **proc syslin** data=sasuser.firms;

aleyland: model i1=mtcap1 nfa1;

mahindra: model i2=mtcap2 nfa1;

tata: model i3=mtcap3 nfa3;

run;

- **proc syslin** data=sasuser.firms sdiag sur;

aleyland: model i1=mtcap1 nfa1;

mahindra: model i2=mtcap2 nfa1;

tata: model i3=mtcap3 nfa3;

run;

Commands

- **proc syslin** data=sasuser.firms sur;

aleyland: model i1=mtcap1 nfa1;

mahindra: model i2=mtcap2 nfa1;

tata: model i3=mtcap3 nfa3;

run;

- **proc syslin** data=sasuser.firms itsur;

aleyland: model i1=mtcap1 nfa1;

mahindra: model i2=mtcap2 nfa1;

tata: model i3=mtcap3 nfa3;

run;

INTERPRETATION

- As market capitalization increases, both Ashok Leyland (AL) and Mahindra (M) invest more whereas Tata (T) invests less which is counterintuitive
- The coefficients of Al and M are significant at 5% level coefficient of T coefficient is insignificant at even 10% level
- As net fixed assets increases, investment by all 3 firms increase, all their coefficients are significant at 5% and 1% level.

Caveats

- $\hat{\beta}_{OLS}$ must be consistent for running iterations and deducing $\hat{\beta}_{FGLS}$
- Prerequisite: no correlation between the error term and explanatory variables
- No endogenity in explanatory variables
- A possible case for first order autoregressive process in error terms

First Order Autoregressive Process

- $\epsilon_{it} = \rho_i \epsilon_{i(t-1)} + v_{it}$ where $|\rho_i| < 1$
- $E(\epsilon_i \epsilon_j') = \Omega_{ij} = \sigma_{ij} (1/1-\rho_i\rho_j) * B$
- $\epsilon_{it} = v_{it} + \rho_i \epsilon_{i(t-1)} = v_{it} + \rho_i [v_{i(t-1)} + \rho_i \epsilon_{i(t-2)}]$ (so on and so forth)
- $\epsilon_{it} = v_{it} + \rho_i v_{i(t-1)} + \rho_i^2 v_{i(t-2)} + \ldots\ldots\ldots + + \rho_i^s v_{i(t-s)} + \rho_i^{s+1} v_{i(t-s-1)} + \ldots$
- B =

$$B = \begin{bmatrix} 1 & \rho_j & \cdots & & \rho_j^{T-1} \\ \rho_i & 1 & \rho_j & \cdots & \rho_j^{T-2} \\ \vdots & & & & \\ \rho_j^{T-1} & \cdots & & & 1 \end{bmatrix}$$

Estimation

- Use $\hat{\beta}_{GLS} = (X' \Omega^{-1} X) X' \Omega^{-1} Y$
- Σ unknown so go for FGLS and iterate
- Estimate β_i and β_j (OLS) → Estimate ϵ_i and ϵ_j → estimate ρ_i, ρ_j → estimate V_{it}, V_{jt} → estimate of σ_{ij}
- Now we get Ω and obtain $\hat{\beta}_{FGLS}$ and set to limit

SUR with Singular Variance Matrices

- General assumption: Variance- Covariance matrix of u_i (Ω) is not singular.
- The above assumption not always true in applications of consumer and producer theory because of additivity constraints
- Example
- $s_{iK} = B_{10} + B_{11} \log p_{iK} + B_{12} \log p_{iL} + B_{13} \log p_{iM} + u_{Ik}$
- $s_{iL} = B_{20} + B_{12} \log p_{iK} + B_{22} \log p_{iL} + B_{23} \log p_{iM} + u_{iL}$
- $s_{iM} = B_{30} + B_{13} \log p_{iK} + B_{23} \log p_{iL} + B_{33} \log p_{iM} + u_{iM}$
- where the symmetry restrictions from production theory have been imposed

- We assume that $E(u_i/p_i)=0$, $u_i=(u_{Ik}, u_{iL}, u_{iM})$, $p_i=(p_{iK}, p_{iL}, p_{iM})$
- $B_{10}+B_{20}+B_{30}=1$
- $B_{11}+B_{12}+B_{13}=0$
- $B_{12}+B_{22}+B_{23}=0$
- $B_{13}+B_{23}+B_{33}=0$
- $u_{iK}+u_{iL}+u_{iM}=0$

- $B_{13}=-B_{11}-B_{12}$
- $B_{23}=-B_{12}-B_{22}$

- $s_{iK}=B_{10}+B_{11}\log(p_{iK}/p_{iM})+B_{12}\log(p_{iL}/p_{iM})+u_{Ik}$
- $s_{iL}=B_{20}+B_{12}\log(p_{iK}/p_{iM})+B_{22}\log(p_{iL}/p_{iM})+u_{iL}$

- Suppose we assume $E\ (u_{ig}/p_{iK}, p_{iL}, p_{iM}) = 0 \quad g=K,L$
- OLS and FGLS estimators are consistent
- System OLS is not OLS equation
- If $Var(u_i/p_i)$ constant, FGLS asymptotically efficient, else use robust variance-covariance matrix estimator for FGLS

Thank You